The Minimal Covering Location Problem with single and multiple location coverage

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***Abstract*— The covering of Location Problems represents very important class of combinatorial problems. One of the most known problem from this class is Maximal Covering Location Problem (MCLP). Its objective is to cover as more as possible location with given facilities. The opposite problem is called Minimal (or Minimum) Covering Location Problem and its aim is to find places for given facilities such as they cover as few as possible location. This paper will present two models of MinCLP derived from the model of classical MCLP.**

I. Introduction

The main objective of covering location problems is in finding optimal positions for facilities with the satisfaction of given conditions. The main parameter of these problems is radius of coverage and it determines an impact of facilities to locations.

One of the most popular covering location problem is Maximal Covering Location Problem (MCLP) and it is introduced by Church and ReVelle [1]. The objective of MCLP is to find positions for the fixed number of facilities in given set of locations so that the total coverage is maximized. This problem is largely applicable in everyday life, such as finding optimal locations of emergency services (ambulance and fire stations), shops, schools etc. MCLP was studied well in past decades and a lot of results have been published. A list of different types of MCLP models follows, but detailed survey from this field could be found in [2]. Moore and ReVelle are introduced a hierarchical service location problem [3], Berman and Krass described gradual covering problem [4] and Qu and Weng described hub allocation maximal covering problem [5]. Capacitated MCLP is defined by Current and Storbeck [6], probabilistic MCLP by ReVelle and Hogan [7] and implicit MCLP is introduces by Murray et al. [8].

On the other hand, Minimal (in literature also known as Minimum) Covering Location Problem is an attempt to minimize the impact of facilities to locations. It is applicable on finding optimal location for undesired facilities, as jails, nuclear and power plants, pollutants etc. This problem has not been much studied in the past, only several papers in this field have been published. Drezner and Wesolowsky [9] presented the Minimum Covering Location problem on the plane with the Euclidean distance between locations. Some researchers studied similar problems, but with different name, like expropriation location problem (ELP) [10]. Exhaustive study of this problem is given by Berman and Huang in [11] and they described five models of Minimum Covering Location Problem with distance constraints (MCLPDC). This paper describes the problem of locating a fixed number of facilities on the network with pre-specified minimal distance between facilities. The objective of MCLPDC is minimization of covered customers.

The aim of this paper is presenting two mathematical models for Minimal Covering Location Problem with single and multiple location coverage. Both models are derived from mathematical model of MCLP presented in [1]. This paper presents theoretical approaches in modelling these problems, without practical methods for finding solutions.

This paper is organized as follows. Mathematical models of MCLP and MCLPDC from [1] and [11] are presented in the section 2. The new approach to modelling Minimal Covering Location Problem are presented in Section 3. Finally, Section 4 briefly summarizes the paper and describes plans for further researches.

II. Mathematical Models of MCLP and MCLPDC

Both problems are defined on given set of locations with pre-defined distances between them. Other constraints in this problems are number of facilities and radius of coverage. The objectives of these problems are locating facilities such as they cover maximum (in MCLP) / minimum (in MCLPDC) locations. Coverage function is determined by radius of coverage.

A. Maximal Covering Location Problem

The original mathematical model of MCLP is described in [1]:

$$\text{maximize} \quad g = \sum_{i \in I} a_i y_i \tag{1}$$

$$\text{subject to} \quad \sum_{j \in N_i} x_j \geq y_i, \forall i \in I \tag{2}$$

$$\sum_{j \in J} x_j = P \tag{3}$$

$$x_j \in \{0,1\}, \forall j \in J \tag{4}$$

$$y_i \in \{0,1\}, \forall i \in I \tag{5}$$

where

- I – set of locations (indexed by i)
- J – set of eligible facility sites (indexed by j)

- S – radius of coverage
- d_{ij} – distance from location *i* to location *j*
- $x_j = \begin{cases} 1, \text{if facility is located at location } j \\ 0, \text{otherwise} \end{cases}$
- a_i – population in node *i*
- P – number of facilities
- $N_i = \{j \in J | d_{ij} \leq S\}$ – set of all facilities *j* which cover location *i*

This model maximizes the sum of covered locations (1) with given conditions (2)-(5). Constraint (2) calculates the coverage of each location y_i - sum on the left side summarizes a number of facilities which cover *i*-th location and determines upper bounds for decision variable y_i. Constraint (3) provides placement of the exact number of facilities and constraints (4) and (5) restrict values of decision variables x_j and y_i.

B. *Minimum Covering Location Problem with distance constraints*

As mentioned before, Berman and Huang [11] described five mathematical models of Minimum Covering Location Problem with distance constraints. We will describe the first model from their research (MCLPDC1) because our model has a similar formulation. Other models in their paper require advanced concepts from graph theory and it is beyond the scope of this paper. Motivation for introduction of distance constraint they described with sensitivity and safety reasons ("if several nuclear reactors are clustered in the same region, they may all be attacked by an aggressor").

The model MCLPDC1 described in [11] is

$$\text{minimize} \quad g = \sum_{i \in I} a_i y_i \quad (6)$$

$$\text{subject to} \quad x_j + x_k \leq 1, \forall j, k \in J \quad (7)$$

$$y_i \geq x_j, i \in I, j \in N_i \quad (8)$$

$$\sum_{j \in J} x_j = P$$

$$x_j \in \{0,1\}, \forall j \in J \quad (9)$$

$$y_i \in \{0,1\}, \forall i \in I \quad (10)$$

where

- S – radius of coverage
- d – minimal distance between facilities
- d_{ij} – distance from location *i* to location *j*
- $x_j = \begin{cases} 1, \text{if facility is located at location } j \\ 0, \text{otherwise} \end{cases}$
- a_i – population in node *i*
- P – number of facilities
- I – set of locations (indexed by *i*)
- J – set of eligible facility sites (sites with minimal distance greater that *d*) (indexed by *j*)
- $N_i = \{j \in J | d_{ij} \leq S\}$ – set of all facilities *j* which cover location *i*

Function g (6) of this problem is the same as in the previous problem (1), but objective of this problem is its minimization. Other constraints determine the fulfillment of all conditions. Constraint (7) state that two locations cannot be located on sites with distance less that *d*. Constraint (8) is similar as constraint (2) and it calculates a coverage of each location y_i. Constraints (8), (9) and (10) are the same as constraints (3), (4) and (5).

III. The New Approach to Modelling Minimal Covering Location Problem

In this paper, Minimal Covering Location Problem will be denote as MinCLP. The main idea of this research is to change model of MCLP with the aim of obtaining the model of MinCLP. This approach, together with introduction of minimal distance constraints, will give modles of two different types of MinCLP. Solutions of both models will be graphic illustrated on the instance with 300 locations, 15 facilities and radius of coverage 4. All locations are generated as points in square with dimensions 30x30 and distances between them are Euclidean distance. Following models will not consider the node population a_i, but this does not reduce the generality of the problem. Population variables are removed order to simplify of the graphic representation of the solution.

Mathematical model of MinCLP obtained from classic MCLP model is follows:

$$\text{minimize} \quad g = \sum_{i \in I} y_i \quad (11)$$

$$\text{subject to} \quad \sum_{j \in N_i} x_j \leq y_i, \forall i \in I \quad (12)$$

$$\sum_{j \in J} x_j = P \quad (13)$$

$$x_j \in \{0,1\}, \forall j \in J \quad (14)$$

$$y_i \in \{0,1\}, \forall i \in I \quad (15)$$

The differences between this model and the MCLP model is in minimization of function g (11) and changing bounds of decision variables y_i (12). Bounds (12) and constraint (15) provide that each location y_i can be covered with at most one facility. It gives an important property of this model – a single coverage for locations. The name of this problem is derived from described properties – Minimal Covering Location Problem with single coverage (MinCLP-SC). Figure 1 illustrates a solution of one instance of MinCLP-SC and it is obvious that each location is covered with at most one facility.

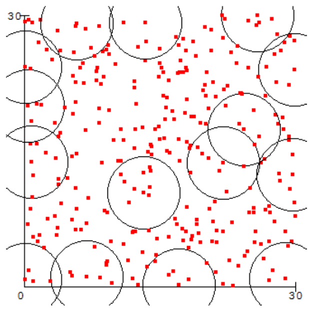

Figure 1. The solution of MinCLP-SC instance of 300 location, 15 facilities and radius of coverage 4

Single coverage is important in modelling problems with this requirement, but the main disadvantage of this model is low solution space. A lot of instances of MinCLP-SC do not have a solution because there is no enough space for placing all facilities with condition of single coverage. In most real-life problems it is necessary to place all facilities, even if they cover more than one location.

The overcoming of this problem can be resolved if the

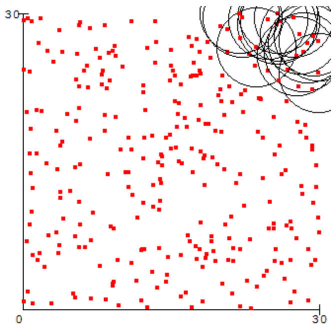

Figure 2. The solution of MinCLP-SC instance of 300 location, 15 facilities and radius of coverage 4

sum from constraint (12) is bounded with value 1. This new constraint allows the covering of each location with more facilities and it gives a Minimal Covering Location Problem with multiple coverage (MinCLP-MC).

Mathematical model of MinCLP-MC is:

$$\text{minimize} \quad g = \sum_{i \in I} y_i \tag{16}$$

$$\text{subject to} \quad \min(1, \sum_{j \in N_i} x_j) \le y_i, \forall i \in I \tag{17}$$

$$\sum_{j \in J} x_j = P \tag{18}$$

$$x_j \in \{0,1\}, \forall j \in J \tag{19}$$

$$y_i \in \{0,1\}, \forall i \in I \tag{20}$$

As mentioned before the difference between MinCLP-SC and MinCLP-MC is in the conditions (11) and (17) and description of this model is not necessary. This type of MinCLP does not have a constraints for a minimal distance between facilities. Figure 2 illustrates a solution of one instance of this type of MinCLP-MC. It is obvious that all facilities are located in near sites and that is not good solution in many real-life problems. Berman and Huang illustrated it on the sensitivity and safety reasons, but this approach is also not good in case of pollutants. Pollution in multiple covered area will be huge and this solution will not be acceptable.

If we add a condition for minimal distance between facilities, we will give a generalization of MinCLP-MC and its mathematical model follows.

$$\text{minimize} \quad g = \sum_{i \in I} y_i \tag{21}$$

$$\text{subject to} \quad \min(1, \sum_{j \in N_i} x_j) \le y_i, \forall i \in I \tag{22}$$

$$\sum_{j \in J} x_j = P \tag{23}$$

$$d_{ij} \ge d, \forall j_1, j_2 \in J, j_1 < j_2 \wedge \tag{24}$$

$$x_{j_1} \cdot x_{j_2} = 1$$

$$x_j \in \{0,1\}, \forall j \in J \tag{25}$$

$$y_i \in \{0,1\}, \forall i \in I \tag{26}$$

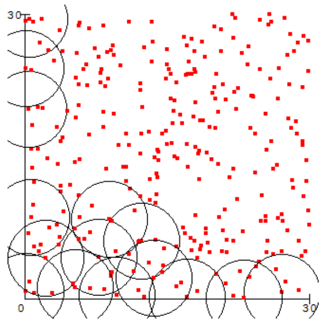

Figure 3. The solution of MinCLP-SC instance of 300 location, 15 facilities and radius of coverage 4 and minimal distance between facilities 4

As mentioned before, this model is generalization of the previous – previous can be obtained with $d = 0$.

The Figure 3 illustrates the solution of the generalized MinCLP-SC.

IV. CONCLUSION

This paper presents a new approach to modelling Minimal Covering Location Problem. This approach is based on conditions of single and multiple location coverage. Two different models are described and graphic representations of their solutions are presented. In the first model, each location is covered with at most one facility and in the second model, location can be covered with more facilities.

In our previous research, we have shown that using fuzzy logic in modelling MCLP significantly improves the quality of models. That will be the main direction for our further work – how to improve MinCLP models with fuzzy conditions.

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