Application of Digital Stochastic Measurement over an Interval in Time and Frequency Domain

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Abstract—Widely employed strategy "measurements in a point" has represented the backbone in measurement evolution and has become a standard method. Time-continuous signals are sampled (at time instant) and converted into discrete digital variables with maximum accuracy. An alternative approach called "digital stochastic measurement over an interval" carries clear advantages in three challenging areas: measurement at high frequencies, measurement of noisy signals and measurement that requires high accuracy and linearity. This overview paper summarizes all cases of application of this approach in time domain as well as in frequency domain which have been developed so far. Described measurement concepts enable simple designing of the instruments with advanced metrological characteristics.

***Keywords*—Electrical measurements, Digital measurements, A/D conversion, Probability, Stochastic processes.**

I. INTRODUCTION

The essence of the sampling method is as follows: in a theoretically infinitely short time interval (practically in an instant), a sample of an analogue measured variable is taken and in a time interval Δt this sample is converted into a number in a device called A/D converter. This commonly used strategy called "measurement in a point" (sampling method measurement) has been the backbone of the measurement instrumentation development in metrology, control, telecommunications, etc. In the conversion process, accuracy and speed are opposing requirements. The mathematics in the background that explains this approach is algebra, while the applied theory is the Theory of discrete signals and systems.

The high speed of all electronic circuits implies the viability of the Central limit theorem for practical measurements. This idea has been known since early 1960'ş [1], but in different context (stochastic computer design). It has been shown [2] that a singular measurement does not need to be maximally accurate, while the measurement uncertainty is reduced by adding a uniform random noise (dither) to the input signal prior quantization (Fig. 1). Probability density function (PDF) of uniform random distribution dither signal h is:

$$p(h) = \frac{1}{a} \text{ for } |h| \le \frac{a}{2} \tag{1}$$

where quantum of uniform quantizer is labeled with $a = 2g$.

Figure 1. Block diagram of application of a uniform random dither h to the measured signal y

The motive of "measurement over an interval" strategy, formulated in [3], was a very simple hardware (low resolution flash A/D converter, hence lowering the number of systematic error sources) and easy parallel processing (practically without additional delays in signal processing). Measurement over an interval is an integral approach to measure a signal and its parameters - a signal [4] or some of its parameters [5] are measured during a finite time interval of an arbitrary duration.

From theoretical point of view, the problem is highly non-linear and stochastic, and therefore neither the standard linear Theory of discrete signals and systems nor the Theory of random processes can be applied. It was necessary to develop an alternative mathematical approach. The time within the measurement interval is treated as a stochastic variable with a uniform distribution. Consequently, the problem of measurement over the interval can be classified in the Probability theory and the area of Statistical theory of sampling.

II. MEASUREMENT IN TIME DOMAIN

A. Measurement of constant voltage

The device called Stochastic additional A/D converter with one dither generator (SAADK-1G) can be used for average value measurement. This device is shown on Fig. 2:

Voltage ranges and decision thresholds associated with process of measuring average input signal by uniform quantizer are represented graphically in Fig. 3.

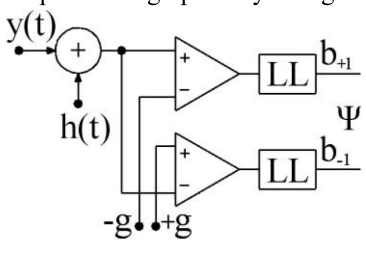

Figure 2. Block diagram of SAADK-1G

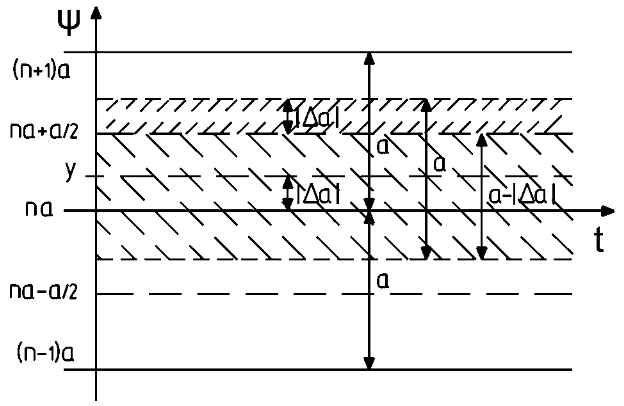

Figure 3. Voltage ranges and decision thresholds associated with process of measuring $\overline{\Psi}$

For the subsequent exposure let us assume that the following conditions are satisfied:

$$|y| \le R,\ \ R = Z \cdot a, \tag{2}$$

Obviously:

$$|y+h| \le R + \frac{a}{2} \tag{3}$$

In the case of dithered constant voltage at the input of the converter $y = const = n \cdot a + |\Delta a|$ let determine average value of the A/D converter's output $\overline{\Psi}$ (from Fig. 1).

$$\overline{\Psi} = \Psi_{n+1} \cdot P(\Psi_{n+1}) + \Psi_n \cdot P(\Psi_n) = n \cdot a + |\Delta a| = y \tag{4}$$

For upper conditions the variance of the variable Ψ is:

$$\sigma_\Psi^2 = (a - |\Delta a|) \cdot |\Delta a| \tag{5}$$

It is obvious that for any $|y| \le R$ the variance is limited with $\sigma_\Psi^2 \le \frac{a^2}{4}$. Consequently, $\overline{\Psi^2}$ is limited as follows:

$$\sigma_\Psi^2 = \overline{\Psi^2} - y^2 \quad \Rightarrow \quad \overline{\Psi^2} \le y^2 + \frac{a^2}{4} \tag{6}$$

This is a very important characteristic of above described measurment process. In this case input signal y is not a constant anymore but rather has its range of value. Fig. 4 shows dependence σ_Ψ^2 of Δa (distance y to nearest quantum level).

Device for average signal value measurement is very simple, with the minimal hardware structure as shown in Fig. 5. As Z (number of positive quantization levels) is not specified, all the above is valid for $Z = 1$. In such a case (2) becomes:

$$|y| \le R,\ \ R = a = 2g \tag{7}$$

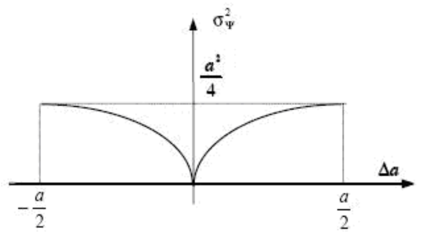

Figure 4. Dependence of output value variance ADC upon distance to nearest quantum level

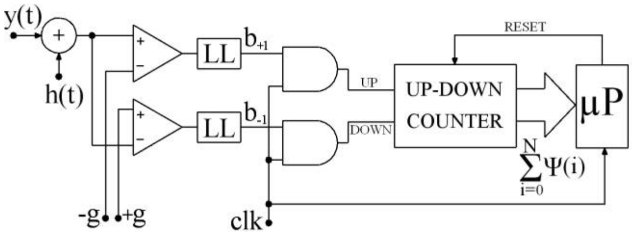

Figure 5. Measurement of the average signal value

obviously:

$$|y+h| \le 3g \tag{8}$$

Possible values of Ψ are $\Psi \in \{-2g, 0, 2g\}$, and the analytical term for Ψ is:

$$\Psi = 2g \cdot (b_1 - b_{-1}) \tag{9}$$

where $b_1, b_{-1} \in \{0,1\}$ and $b_1 \cdot b_{-1} = 0$. It is never possible that b_1 and b_{-1} are equal to 1 simultaneously - it would have meant that $y \ge 0$ and $y \le 0$ simultaneously.

The counter shown on Fig. 5 works like accumulator - it accumulates summands $(b_{1i} - b_{-1i})$. At the end of measurment interval accumulator output value is equal to $\sum_{i=1}^{N} (b_{1i} - b_{-1i})$. Microprocessor gets information about number of samples from clock. At the end of measurement interval microprocessor calculates average value $\overline{|\Psi|}$ as:

$$\overline{\Psi} = \frac{2g}{N} \cdot \sum_{i=1}^{N} (b_{1i} - b_{-1i}) = \frac{1}{N} \cdot \sum_{i=1}^{N} \Psi_i = y \tag{10}$$

If the sampling frequency tends to infinity and the registers are of infinite length, then $\overline{\Psi} = y$. This relation is also valid if the sampling frequency is finite, but the measurement time is infinitely long.

B. Measurement of the average value of limited function over closed time interval

The crucial question for practical implementation of the measurement shown at Fig. 5 is what is the difference between $\overline{\Psi}$ and y if both, sampling frequency and the measurement interval, are finite? For assessing the measurement uncertainty for the measurement from Fig. 5 we need to apply Central limit theorem.

This result introduces the measurement time interval, not a point in time, as an important variable for expressing measurement results.

Let $y = f(t)$ (it is not limited to be constant signal) be a limited integrable function and h a uniform random signal, both of which satisfy conditions given with (1). If t is a random variable with a uniform distribution whose PDF is: $p(t) = 1/(t_2 - t_1)$, then y is random variable dependent on t. Then the average value of the A/D converter's output $\overline{\Psi}$ over the interval $t \in [t_1, t_2]$. (applying of sifting property of Dirac delta functions) is given as:

$$\overline{\Psi} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f(t) \cdot dt = \overline{y} \tag{11}$$

This result introduces the measurement time interval, not a point in time, as an important variable for expressing measurement results. The device from Fig. 5 measures average value of the input signal over the interval.

If $Z=1$, if error e of a single measurement of y is defined with $e=\Psi-y$, then variance of the error $\sigma_e^2=\sigma_\Psi^2-\sigma_y^2$ of this measurement is:

$$\sigma_e^2=\frac{2g}{t_2-t_1}\int_{t_1}^{t_2}\left|f(t)\right|\cdot dt-\frac{1}{t_2-t_1}\int_{t_1}^{t_2}f^2(t)\cdot dt \tag{12}$$

Quantity e is random, and $\bar{e}=0$, hence $\overline{e^2}=\sigma_e^2$.

Because both the Central limit theorem and Statistical sampling theory can be applied to the individual measurements error e, average value of the A/D converter's output is:

$$\bar{\Psi}=\bar{y}+\bar{e} \tag{13}$$

Because $\bar{\Psi}=\bar{y}$, average error is $\bar{e}=0$, hence:

$$\sigma_{\bar{\Psi}}^2=\sigma_{\bar{y}}^2+\sigma_{\bar{e}}^2 \tag{14}$$

As y is deterministic variable that characterizes the signal, thus having error determined only with member $\sigma_{\bar{e}}^2$ (not with $\sigma_{\bar{y}}^2$).

Also, as Central limit theorem and statistical sampling theory can be applied to the error e, next estimation follows:

$$\sigma_{\bar{e}}^2=\frac{\sigma_e^2}{N} \tag{15}$$

where N is the number of samples within the time interval $T=t_2-t_1$.

More details on this measurement case are given in [3, 9, 10] and according to them standard measurement uncertainty can be defined as $\sigma_{\bar{e}}$.

C. Measurement of amplitude and RMS value the sinusoidal signal

If we use device from Fig. 5 to measure sinusoidal signal $y(t)=A\cdot\sin\omega t$ average value, measurement value will always be 0, because the average value of sine or cosine signals on integer number of periods is equal to 0.

For this purpose the structure of device from Fig. 5 is modified by adding one "or" circuit together with using „up" counter input only so providing „a two-way rectifier". Microprocessor gets number of periods from "Zero Crossing Detector" (ZCD) - Fig. 6.

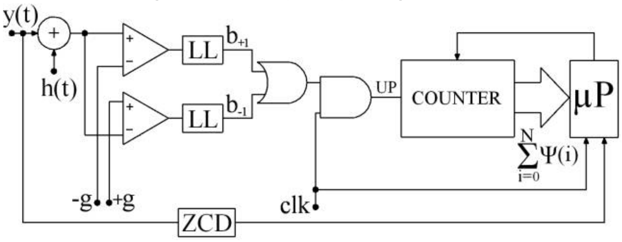

Figure 6. Implementation of sinusoidal signal measuring device based on modified SAADK-1G

Ψ takes its values from $\Psi\in\{0,\,1\}\equiv\{0,\,2g\}$, so the converter output is now given with:

$$\overline{|\Psi|}=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\left|f(t)\right|dt \tag{16}$$

Corresponding measurement error is:

$$\sigma_{\overline{|\Psi|}}^2=\frac{\dfrac{2g}{t_2-t_1}\displaystyle\int_{t_1}^{t_2}\left|f(t)\right|dt-\left[\dfrac{1}{t_2-t_1}\displaystyle\int_{t_1}^{t_2}\left|f(t)\right|dt\right]^2}{N} \tag{17}$$

In the case of sinusoidal signal $y(t)=A\cdot\sin\omega t$ converter output is:

$$\overline{|\Psi|}=\frac{1}{T}\int_0^T\left|A\cdot\sin\omega t\right|dt=\frac{A}{T}\cdot\frac{T}{2\pi}\cdot(1+1)=\frac{2A}{\pi} \tag{18}$$

For each individual analytic sample Ψ is $\Psi_i=2g\cdot(b_{1i}-b_{-1i})$. The counter shown on picture is now again in role of accumulator, but now it accumulates summands $\left|b_{1i}-b_{-1i}\right|$. At the end of measurement interval accumulator output value is equal to $\sum_{i=1}^{N}\left|b_{1i}-b_{-1i}\right|$. Microprocessor gets information about number of samples from clock. At the end of measurement interval microprocessor calculates average value $\overline{|\Psi|}$ as:

$$\overline{|\Psi|}=\frac{2g}{N}\cdot\sum_{i=1}^{N}\left|b_{1i}-b_{-1i}\right|=\frac{1}{N}\cdot\sum_{i=1}^{N}\left|\Psi_i\right|=\overline{|y|} \tag{19}$$

Modified SAADK-1G can be used for sinusoidal signal $y(t)=A\cdot\sin\omega t$ RMS measurement as described before. Measurement accuracy is then:

$$\sigma_{\overline{|\Psi|}}^2=\frac{2g\cdot\dfrac{2A}{\pi}-\dfrac{4A^2}{\pi^2}}{N}=\frac{4A}{\pi^2 N}\left(g\pi-A\right) \tag{20}$$

Depedence of output value variance (which is also the absolute measurement error) is shown on Fig. 7.

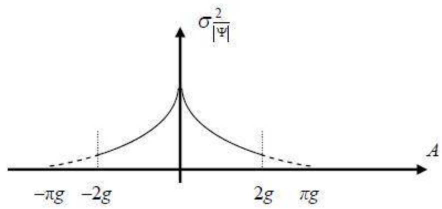

Figure 7. Error diagram for modified SAADK-1G in sinusoidal regime

D. Measurement of RMS value of complex periodic signal

Device for this measurement is called Stochastic additional A/D converter with two dither generators (SAADK-2G). It is shown on Fig. 8.

When performing this measurement the same signal is introduced to both device inputs $y_1(t)=y_2(t)=u(t)$ and RMS value of the signal is calculated as:

$$U_{eff}=\sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt} \tag{21}$$

Device working on this base is called Stochastic additional A/D converter with two dither generators (SAADK-2G) as shown on Fig. 8.

This measurement is practically reduced to the special case of power measurement.

E. Measurement of power

SAADK-2G device shown on Fig. 8 is used also for active power measurment. We have voltage signal $y_1(t) = u(t)$ on the one imput of SAADK-2G and current signal $y_2(t) = i(t)$ on the other one. Power of signal is now calulated as:

$$P = \frac{1}{T}\int_0^T u(t)\cdot \mathrm{i}(t)\,dt \tag{22}$$

Device from Fig. 8 have 2 two-bit flash A/D converters from Fig. 5, with inputs $y_1 = f_1(t)$ and signal h_1, and $y_2 = f_2(t)$ and signal h_2, respectively. h_1 and h_2 are mutually uncorrelated random uniform dither signals. Outputs Ψ_1 and Ψ_2 are passed to a multiplier; the multiplier output is $\Psi = \Psi_1 \cdot \Psi_2$, and it can assume values: $\Psi \in \{-(2g)^2, 0, +(2g)^2\}$.

If, during one measurement interval, N A/D conversions are performed by each A/D converter, then the accumulator from Fig. 8 accumulates the sum of N subsequent multiplier's outputs: $\sum_{i=1}^{N}\Psi_1(i)\cdot\Psi_2(i)$.

This accumulation can be simply used for calculation of the average value of the multiplier output $\overline{\Psi}$ over the measurement interval as:

$$\overline{\Psi} = \frac{1}{N}\cdot\sum_{i=1}^{N}\Psi_1(i)\cdot\Psi_2(i) \tag{23}$$

Over the time interval $T = t_2 - t_1$, the average value of the multiplier output $\overline{\Psi}$, is determined as follows:

$$\overline{\Psi} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f_1(t)\cdot f_2(t)\cdot dt = \overline{y_1 \cdot y_2} \tag{24}$$

The practical consequence of this is that device from Fig. 8 can be used for measurement of signal power.

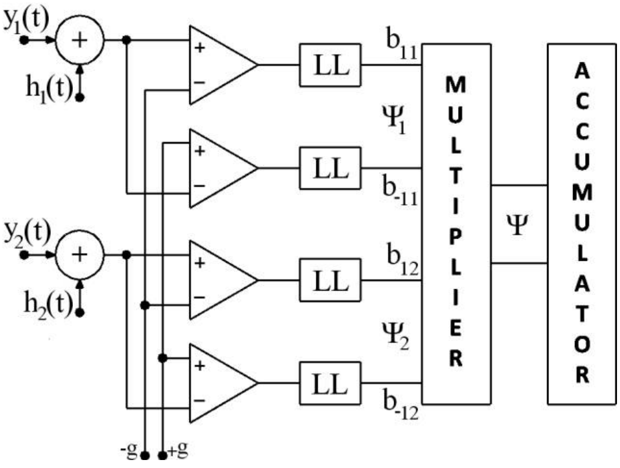

Figure 8. Block diagram of SAADK-2G for realization of RMS measuremet

Let's determine the variance of measurement error for product of signals y_1 and y_2 when signals power is measured using the device from Fig. 8. If $\Psi = y_1 \cdot y_2 + e$ is an instantaneous value of the multiplier output, where e is the product's measurement error, the variance $\sigma_e^2 = \sigma_\Psi^2 - \sigma_{y_1 y_2}^2$ of the error e is:

$$\sigma_e^2 = \frac{(2g)^2}{t_2 - t_1}\int_{t_1}^{t_2}\left|f_1(t)f_2(t)\right|dt - \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f_1^2(t)f_2^2(t)dt \tag{25}$$

The consequence of above, is that the Central limit theorem and the theory of samples are both valid for the quantity e, hence:

$$\sigma_{\bar{e}}^2 = \frac{\sigma_e^2}{N} \tag{26}$$

where N is the number of samples within the time interval $T = t_2 - t_1$.

Estimation (26) is correct if the discrete sets of samples $\Psi_1 \in \{\Psi_1(1), \Psi_1(2), \cdots, \Psi_1(N)\}$ can represent function $y_1 = f_1(t)$ and $\Psi_2 \in \{\Psi_2(1), \Psi_2(2), \cdots, \Psi_2(N)\}$ can represent function $y_2 = f_2(t)$, which means that the Nyquist's conditions, regarding a uniform sampling of the signals y_1 and y_2, is satisfied.

The limit of precision is analyzed theoretically, via simulation and experimentally in [5, 7].

F. Measurement of definite integral product of two or more signals

The device from Fig. 8 can be extended to S inputs ($y_i = f_i(t), i = 1,2,..., S$), and the two-input multiplier can be replaced with a *S*-input multiplier, thus being adjusted for measurement of the averaged multiplier's output as follows:

$$\overline{\Psi} = \frac{1}{N}\cdot\sum_{i=1}^{N}\Psi_1(i)\cdot\Psi_2(i)\cdot\ldots\cdot\Psi_S(i) \tag{27}$$

This can be generalized for a product of S signals, and in such a case, the averaged output of the multiplier is also:

$$\overline{\Psi} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f_1(t)\cdot f_2(t)\cdot\ldots\cdot f_S(t)\cdot dt \tag{28}$$

Variation of the measurement error is:

$$\begin{aligned}\sigma_e^2 = {} & \frac{(2g)^S}{t_2 - t_1}\int_{t_1}^{t_2}\left|f_1(t)\cdot f_2(t)\cdot\ldots\cdot f_S(t)\right|\cdot dt - \\ & - \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} f_1^2(t)\cdot f_2^2(t)\cdot\ldots\cdot f_S^2(t)\cdot dt\end{aligned} \tag{29}$$

III. MEASUREMENT IN FREQUENCY DOMAIN

The RMS instruments in [8] and [9] measure the same input signal in two internal channels in low resolution but at a very high speed. Two uncorrelated stochastic dither functions are superimposed onto input signals in channels 1 and 2, as shown in Fig. 9. The high speed of both sampling and further processing facilitates the RMS measurement by the very definition of the RMS value of a signal, giving a higher resolution of the result and, hence, very accurate measurements.

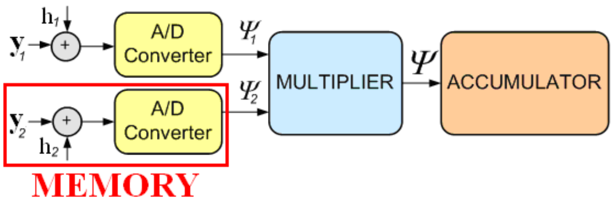

Figure 9. Schematic of the stochastic instrument for one (sine or cosine) harmonic component

As the aim of the instrument presented in this paper is to measure the harmonics of mains voltages and currents, the RMS instruments in [8] and [9] are adapted for such a purpose. The first modification is that the input to channel 2 is not a measured signal but a dithered sine or cosine function that is generated in advance, stored in the MEMORY, and quickly retrieved during measurement, as shown in Fig. 9. Second, such a structure is implemented in parallel for each sine and cosine component of each harmonic to be measured, thus enabling very high processing speed (Fig. 10).

Dithering signals h_1 and h_2 are random, uniform, and mutually uncorrelated [2]. They are generated in a way to satisfy the following conditions that limit their amplitude and define their probability density function:

$$0 \le |h_i| \le \Delta_i / 2 \tag{30}$$

$$p(h_i) = 1/\Delta_i, \quad \text{for } i = 1,2 \tag{31}$$

They are also superimposed onto the measured signals y_1 (any continuous function of time) and y_2 (a continuous base function in the generalized case or a prestored dithered sine or cosine function in a specific case of Fig. 9), respectively. Their sampled values at every time instant within measurement interval T are Ψ_1 and Ψ_2, respectively. These sums are then processed by two flash A/D converters, which perform the A/D conversion within a single clock cycle. The A/D converters can be of a two-bit structure, as in [8], or a multi-bit structure in the generalized case.

The sampled digital signals Ψ_1 and Ψ_2 are multiplied, and their product Ψ is numerically integrated in the accumulator over measurement period T. Finally, a microprocessor (not shown in Fig. 9) will pick up the accumulated value at the end of the measurement interval to perform the final processing.

The measured value (i.e., multiplier output) Ψ differs from the input signal's product by measurement error e, which stems from a combined effect of quantization within the A/D converter and the introduced dither, i.e:

$$\Psi = \Psi_1 \cdot \Psi_2 = y_1 \cdot y_2 + e \tag{32}$$

The first term of this multiplier output is the signal that has to be measured. The first and the second terms in (32) are independent; hence, their average values over the measurement period and their variances are also uncorrelated. The average value of the second term in (32) is zero, as shown in [9], and hence does not feature in the average value of the expected output $\overline{\Psi}$ over the measurement period. Hence:

$$\overline{\Psi} = \frac{1}{T}\int_0^T y_1 \cdot y_2 \, dt \tag{33}$$

In digital measurements, the average value is obtained as

$$\overline{\Psi} = \frac{1}{N}\sum_{k=1}^{N} \Psi_k \tag{34}$$

The summing of the samples during the measurement interval is done by the instrument itself, and this sum is the output of the structure shown in Fig. 9. The division by the number of samples N is performed in a microprocessor, which also calculates each sine (or cosine) component of the k-th harmonic from the appropriate channel as:

$$a_k = 2\overline{\Psi}_{\cos k} / R, \qquad b_k = 2\overline{\Psi}_{\sin k} / R \tag{35}$$

The calculation of measurement uncertainty is an extension of the theory developed in [6]. Relative measurement uncertainty for the digital multiplier output $\overline{\Psi}$ is limited by:

$$u(\overline{\Psi}) \le (Y_2 \cdot \frac{\Delta_1}{2}) / \sqrt{N} \tag{36}$$

$$u \le (Y_2 \cdot \frac{\Delta_1}{2}) / (\overline{\Psi} \cdot \sqrt{N}) \tag{37}$$

where Y_2 is the RMS value of the dithered base function signal in channel 2, Δ_1 is the A/D converter quantum, and N is the number of samples over a measurement interval [6]. The relative standard uncertainty is related to the standard measurement uncertainty $u(\overline{\Psi})$ by:

$$u = u(\overline{\Psi}) / \overline{\Psi} \tag{38}$$

If R is the amplitude of the dithered base function Y_2, then:

$$Y_2 = R / \sqrt{2} \tag{39}$$

According to (36), (37), (38) and (39) and the relation between the Fourier coefficients a_k and b_k and $\overline{\Psi}$ presented in [12], the standard measurement uncertainty of any Fourier coefficients measured with this method is limited by:

$$u(a_k) = u(b_k) \le \sqrt{2} \cdot \frac{\Delta_1}{2} / \sqrt{N} \tag{40}$$

and standard measurement uncertainty of harmonic amplitude is given by:

$$u(\sqrt{a_k^{\,2} + b_k^{\,2}}) \le \Delta_1 / \sqrt{N} \tag{41}$$

Therefore, the system can have very good accuracy when increased number of samples N. If the A/D

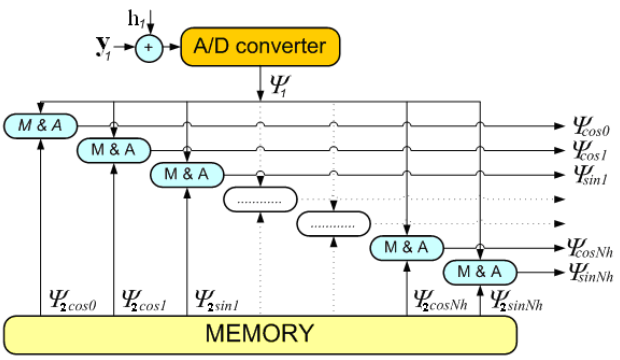

Figure 10. Schematic of the stochastic instrument for one (sine or cosine) harmonic component

converter would be an ideal one, then $\Delta_1 = 0$, and the right side of (40) was 0.

It can be seen that the measurement uncertainty is influenced by the RMS value of the signal in channel 2 Y_2 and the resolution in channel 1 Δ_1, as well as by the number of samples within the measurement period N.

At first sight, it seems that the reduction in Y_2 will reduce the measurement uncertainty. However, Y_2 also (via Ψ_2 and Ψ) defines the value of $\overline{\Psi}$; hence, the measurement uncertainty (41) is not sensitive to the amplitude of the signal in channel 2. On the contrary, Y_2 should be as large as possible, so that the channel-2 range is fully utilized, thus maximizing the measurement result $\overline{\Psi}$. Signal y_2 is within the range $\pm R$; quantum Δ_1 is defined by the chosen A/D converter resolution in channel 1, whereas the number of samples N can be a compromise between the necessary instrument speed and the required accuracy.

Measurement uncertainty, prototype devices and experimental results of this measurement case are discussed in [4, 6].

CONCLUSION

This is an overview paper which summarizes all cases of application of “digital stochastic measurement over an interval” approach in time domain as well as in frequency domain, which have been developed so far. Described measurement systems enable simple designing of the instruments with advanced metrological characteristics.

Several prototypes and small-series of commercial instruments have been developed and their measurement uncertainty is being kept extremely low [7-9]. Fig. 11 shows one practical implementation of the device from Fig. 8 - the four-channel three-phase power analyzer [11], that works on the principles elaborated in the paper. The unit measures 16 values of RMS of current, 3 values of RMS of voltage, distortion factor of three-phase voltage, 12 active powers, 3 frequencies, and over a hundred derived values. All values have variable resolution in time and value. The basic accuracy of these values is 0.2% of full scale. This device is remotely controlled.

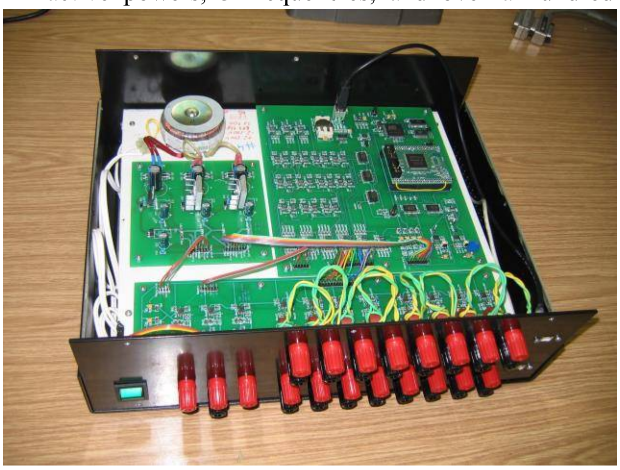

Figure 11. One practical implementation of the device - the four-channel three-phase power analyzer

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