Fuzzy Influence Diagrams in Power Systems Diagnostics

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Abstract — **In this paper, influence diagram with fuzzy probability values, as a graphical tool for the diagnostic reasoning in power system has been proposed. Instead of Bayesian networks that are using conditional probability tables, often difficult or impossible to obtain, a verbal expression of probabilistic uncertainty, represented by fuzzy sets is used in this paper. The proposed methodology enables both type of inference support: bottom-up and top-down inference, including decision nodes in the analysis. This inference engine is illustrated on the case of the detection of the cause of excessive tripping of transformer breaker in the substation.**

I. INTRODUCTION

Problems of diagnostics in power systems, including the detection of failures, or equipment condition assessment are faced with many uncertainties related to their past, present and future operational conditions. Bayesian networks and Influence diagrams are graphical tools that aid reasoning and decision-making under uncertainty, modeling the system with the network of states with known probability distributions [7,11,13]. These tools are used for the medical diagnosis, map learning, heuristic search, and, very recently, in power systems, including both the predictive and diagnostic support.

In power systems, the predictive support is used mostly for the prediction of circuit breaker or transformer failures [4, 17] based on condition monitoring data. Second approach – fault diagnostic is applied for relay protection selectivity and transformer fault diagnostic [3, 18].

Using probabilistic methods for the linking of symptoms to failures is possible only in the presence of necessary failure probabilities, obtained from operating data, or through the solicitation of subjective probabilities from experts. However, this is not always possible, and depends on quality and quantity of available data.

The objective of this work is to propose an integrated method for both types: bottom-up and top-down inference support in uncertain environment, including decision nodes in the analysis. The rest of the paper is organized as follows. Section II discusses basic concept of ID modeling. Section III gives details of the fuzzy influence diagram model, while Section IV elaborates the case study: the determining of the cause of excessive tripping of transformer circuit breaker.

II. INFLUENCE DIAGRAMS

Influence diagrams were proposed by Howard and Matheson [7], as a tool to simplify modeling and analysis of decision trees. They are graphical aid to decision making under uncertainty, which depicts what is known or unknown at the time of making a choice, and the degree of dependence or independence (influence) of each variable on other variables and choices. It represents the cause-and-effect (causal) relationships of a phenomenon or situation in a non-ambiguous manner, and helps in a shared understanding of the key issues.

Building of an influence diagram is performed with the usage of several graphical elements. A circle depicts an external influence (an exogenous variable), rectangle depicts a decision. Chance node (oval) represents a random variable whose value is dictated by some probability distribution, and value node is presented as a diamond (objective variable) - a quantitative criterion that is the subject of optimization.

The diagram can be used as a basis for creating computer-based models that describe a system or as descriptions of mental models managers use to assess the impact of their actions. Influence diagram represents a pair $N=\{(V, E), P\}$ where V and E are the nodes and the edges of a directed acyclic graph, respectively, and P is a probability distribution over V. Discrete random variables $V=\{X_1, X_2, ..., X_n\}$ are assigned to the nodes while the edges E represent the causal probabilistic relationship among the nodes. Each node in the network is annotated with a Conditional Probability Table (CPT) that represents the conditional probability of the variable given the values of its parents in the graph. However, the use of probability tables with many elements is very difficult, because of the combinatorial explosion arising from the requirement that the solution must be extracted by the cross product of all probability tables.

Solving of an ID can be effectuated using fuzzy reasoning [1, 8, 9, 12], where each node in the diagram can be represented by appropriate fuzzy sets, describing the uncertain nature of a given value. The combination of predecessor nodes fuzzy sets gives the value of resulting node. A commonly used technique for combining fuzzy sets is Mamdany's fuzzy inference method. However, the main limitations of fuzzy reasoning approaches are the lack of ability to conduct inference inversely. Feed-forward-like approximate reasoning approaches are strictly one-way, that is, when a model is given a set of inputs can predict the output, but not vice versa.

Furthermore, utilization of a probability measure to assess uncertainty requires too much precise information in the form of prior and conditional probability tables, and such information is often difficult or impossible to obtain. In certain circumstances, a verbal expression or interval value of probabilistic uncertainty may be more appropriate than numerical values. The fuzzy influence diagram with the fuzzified probabilities of states is presented in the next section.

III. FUZZY INFLUENCE DIAGRAMS

The fuzzification of influence diagrams used in this approach is performed both by the fuzzification of random variables, like in [15, 16], and by introduction of fuzzy probabilities [5, 6, 10]. Based on previous works on linguistic probability [5, 6], it is possible to define similar probability measure for fuzzy probabilities.

Definition 1. Given an event algebra ε defined over a set of outcomes Ω, a function *FP*: ε→E is termed a fuzzy probability measure if and only if for all $A \in \varepsilon$

$$0_\chi \circ FP(A) \circ 1_\chi \tag{1}$$

$$FP(\Omega) = 1_\chi \; and \; FP(\varnothing) = 0_\chi \tag{2}$$

$$If \; A_1, A_2, \ldots \in \varepsilon \; are \; disjoint, then$$

$$FP(\bigcup_{i=1}^{\infty} A_i) \subseteq \sum_{i=1}^{\infty} FP(A_i) \tag{3}$$

where *FP* is fuzzy probability measure on (Ω, ε), the tuple (Ω, ε, *FP*) is termed fuzzy probability space.

Embedded real numbers are denoted by χ subscript. Based on previous definition, fuzzy probabilities, grouped in several fuzzy sets, are introduced and denoted with linguistic terms (extremely low, very low, low, medium low, medium, medium high, high, very high and extremely high) and presented on Figure 1.

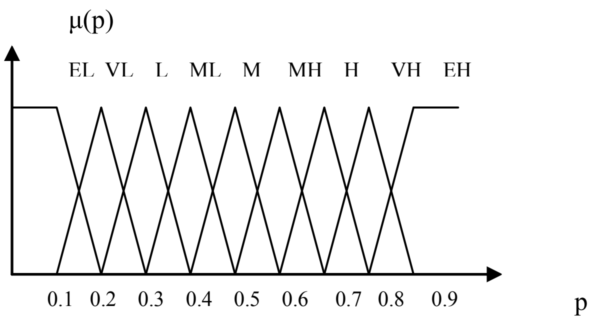

Figure 1 Fuzzy probabilities

From previous definition, two fuzzy Bayes rules analogue to classical crisp number relations (4) and (5) are formulated. Operator "≅" stands for "=" operator.

$$\begin{aligned} &FP(Y = y_j, X = x_i) \cong \\ &\cong FP(X = x_i) \otimes FP(Y = y_j \setminus X = x_i) \end{aligned} \tag{4}$$

$$\begin{aligned} &FP(X = x_i \setminus Y = y_j) \cong \\ &\cong \frac{FP(X = x_i) \otimes FP(Y = y_j \setminus X = x_i)}{FP(Y = y_j)} \end{aligned} \tag{5}$$

Based on the law of total probability another rule for the fuzzy marginalization can be added, represented by the expression (6).

$$\begin{aligned} &FP(Y = y_j) \cong \\ &\cong \sum_i FP(X = x_i) \otimes FP(Y = y_j \setminus X = x_i) \end{aligned} \tag{6}$$

Using the above equations, fuzzy Bayes inference can be conducted, with operations of fuzzy numbers defined as operations in terms of arithmetic operations on their α – cuts (arithmetic operations on closed intervals).

The main advantage of the proposed method is very flexible operation with uncertain data, which are now presented in verbal form, through fuzzy inference rules. Prior and conditional probabilities for individual nodes are presented in tables, and they are following fuzzy inference rules derived from expert knowledge base. The elements of matrix $M=[M_{ij}]$, where *m* is the number of discrete states of parent node X_i, and *n* is the number of discrete states of the child node Y_j represent possible state of the nature, denoted with the name of node (A, B, C...) and the number of node state (I, II, III). M_{ij} represents the conditional probability $FP(Y = y_j, X = x_i)$.

This methodology will be illustrated on the case of the power transformer diagnostics.

IV. CASE STUDIES

The methodology for both predictive and diagnostic support is illustrated on the case of power transformer in one transformer substation, planned for the replacement, because of its age and unsatisfying diagnostic test results. Transformer deterioration is modeled with three deterioration stages, with parameters represented in table I.

TABLE I
DECISION NODE A

Decision	Description
I	Replace
II	Do Nothing

The illustration of influence diagram for the transformer risk assessment is given on figure 2.

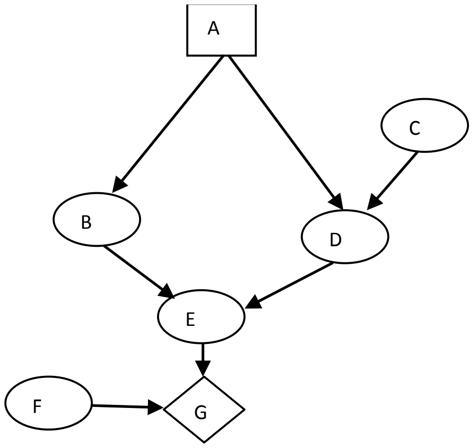

Figure 2. Transformer condition assessment

A – decision node
B – transformer condition
C – wheather conditions
D – transformer loading
E – failure probability
F – penalty policy
G – value of risk node

Node A is decision node, and decision is bound to only two decisions: whether to replace (AI) or keep the existing power transformer in use (AII).

A. Predictive support

Increased number of transformer outages is expected, but that number, together with consequences that these outages will produce can vary depending on uncertain parameters in the future, including weather conditions, loading of the transformer, and level of penalties imposed by the regulator. Therefore, one has to investigate the possibility of keeping it in service one more year, and to check whether this decision greatly increases the risk of surpassing required values for system reliability, imposed by the regulator.

Node B is describing the condition of transformer itself. It can be described by three deterioration states: bad, medium and good. This node represents the chance node, because the condition of the transformer is of the stochastic nature, and cannot be fully determined by transformer diagnostic. On the other hand, this node has the parent node A, because the state of transformer's health is directly influenced by the decision of replacement. The conditional probabilities are presented in Table II.

Node C is independent node, describing future weather conditions of purely stochastic nature (Table III).

TABLE II
CHANCE NODE B TRANSFORMER CONDITION

State	Description	Failure rate λ (int./year)	Probability	
			AI	AII
I	Good	0,02	H	EL
II	Medium	0,05	VL	L
III	Bad	2	EL	MH

TABLE III
CHANCE NODE C AMBIENT CONDITIONS

State	Description	Probability
I	Moderate temperature, not below -30	H
II	Severe conditions, temperature below -30	L

Node D is the chance node describing the loading of the transformer. This node has two parent nodes: in the case of cold winter, the loading will increase. The decision of keeping the existing transformer will also affect the loading, because in the case of replacement, dispatcher would more likely decide to put more loads to the new transformer from surrounding feeders (Table IV).

TABLE IV
CHANCE NODE D TRANSFORMER LOADING

State	Descriptio n	Conditional Probabilities			
		AI	AI	AII	AII
		CI	CII	CI	CII
I	Below maximum	H		H	EL
II	Around maximum	VL	L	VL	L
III	Above maximum	EL	H	EL	MH

Node E is chance node describing reliability parameters depending on related transformer station (Table V). The calculation of expected failure probability, and consequently the SAIFI parameter is based on Poisson law, with λ denoting the failure rate from table II, and *k* representing the number of failures.

$$f(k) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{7}$$

The usual metrics of system reliability is System Average Interruption Index (SAIFI). Hypothetically, the new law, which will drastically increase the penalties in the case of surpassing value of 2 interruptions per customer and year, is expected, but some uncertainty about the date of adoption still exists. Verbally modeled probabilities are presented in table VI.

TABLE V
CHANCE NODE E

State	Description	Conditional Probabilities								
	SAIFI	BIDI	BIDII	BIDIII	BIIDI	BIIDII	BIIDIII	BIIIDI	BIIIDII	BIIIDIII
I	<1	EH	VH	H	MH		EL	VL	VL	
II	1-2			L	L	ML	L	ML	L	ML
III	>2	EL	VL		EL	MH	MH	ML	M	MH

TABLE VI
CHANCE NODE F PENALTIES

State	Description	Probabilities
I	New energy law adopted, severe penalties – [0.8 0.9 1]	H
II	New law not yet adopted, mild penalties [0.2 0.3 0.4]	L

The value node G is the risk node, defined as the product of probability and consequence (financial penalty).

In this simplified model of power transformer, its condition can be assessed by two independent variables: Age of the transformer and the Furan content (FC). Both *Age* and *FC* can be represented by triangular fuzzy sets, with following presumed membership functions: *Age* (Young[0 0 15], Medium [5 25 40], Old [25 40 40]); *Furan content*(Low [0 0 2000], Medium [0 2000 4000], High [2000 4000 4000]) with variables expressed in years and ppm, respectively. The condition of the transformer will be represented by three states: Good, Medium and Bad.

To calculate the probability of transformer being in one of deterioration states, the results of diagnostic tests of the furan content *FC*, which is directly influenced by the loading history of the transformer is used (Figure 3). Conditional probabilities of deterioration state, depending on the decision (or the age of the transformer) are presented in table III and are expressed by appropriate fuzzy sets.

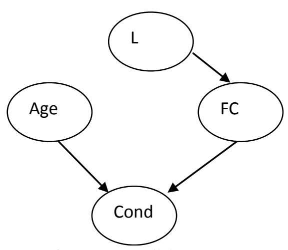

Figure 3. Transformer condition assessment

Chance node *L* is the parent node to chance node *FC*, and node *Cond* is the child node for both *Age* and *FC* nodes. If both *Age* and *L* are represented by discrete nodes, rules for probability calculations of child node and parent nodes are presented in equations (8) –(10).

$$P\left(Cond_i, Age, FC\right) = \\ = \sum_j \sum_k P\left(Cond_i \setminus Age, FC\right) \otimes P(Age_j) \otimes P(FC_k) \tag{8}$$

$$P\left(FC_k\right) = \sum_j P\left(FC_k \setminus L_j\right) P(L_j) \tag{9}$$

Probability that, given the evidence that condition is in the state *i*, hypothesis of loading being in the state *j* is:

$$P\left(L_j \setminus Cond_i\right) = \frac{P\left(L_j\right) P(Cond_i \setminus L_j)}{P(Cond_i, Age, FC)} \tag{10}$$

For the transformer that is, for example, 25 years old, with the furan content of 2200 ppm, we are getting following membership functions: *Age* = (0 young, 0,4 medium 0,4 old) and *FC* = (0 low, 0,95 medium 0,05 high). For the sake of practical representation of data in BN, a simple mapping of random variable to appropriate fuzzy probability $X_i \rightarrow FP(X_i)$ has to be performed, by the selection of appropriate fuzzy probability set . For the proposed example, mapping is presented in the following table.

TABLE VII
MAPPING OF FUZZY VARIABLES TO FUZZY PROBABILITY MEASURES

Variable: Age	Fuzzy probability	Variable: Furan content	Fuzzy probability
Young	EL	Low	EH
Medium	M	Medium	M
Old	M	High	EL

The value node, G, represents the risk associated to particular event *E*. *FP(E)* is fuzzy probability calculated for the node *E*, and final value of risk is the expected value of risk for all combinations of event *E* over *N* possible outcomes of event *F*. PEN_i denotes penalties in the case of the *i*-th outcome of event *F*. Penalties are also represented as fuzzy numbers, and they are given in per unit values, relative to the maximal possible penalty:

$$Risk = \sum_{i=1}^{N} FP(E) \otimes FP(F_i) \otimes PEN_i \tag{11}$$

Using expressions for the fuzzy joint probability and Bayes rule, we are calculating the value of node G (Figure 4). Different methods of fuzzy number ordering can be used, and final results are showing that by replacing the transformer we are reducing the risk more than two times [9].

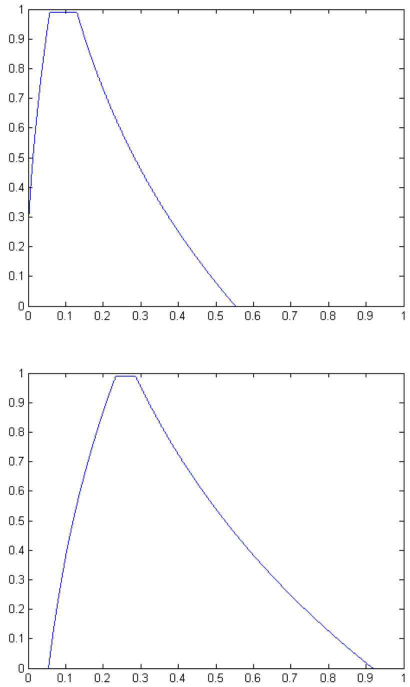

Figure 4. Fuzzy values of risk node G for two alternative decisions for a) replacing and b) keeping the existing transfomer

B. Diagnostic support

In the case of diagnostic support, we will presume that results from previous year show the level of SAIFI surpassing 2 interruptions per customer. The cause of interruptions are unknown (internal fault in the transformer followed by Buholtz relay tripping, contact thermometer, or overcurrent relay tripping caused by the overloading). The transformer in supplying transformer station has not been replaced (AII), but its condition is unknown. Weather conditions were severe (CII). To calculate the probability that condition of transformer is good (BI) in spite of achieved level of reliability, the expression (6) is used.

$$FP(B = BI \setminus E = EIII) \cong$$
$$\cong \frac{FP(B = BI) \otimes FP(E = EIII \setminus B = BI)}{FP(E = EIII)} \quad (8)$$

Obtained fuzzy numbers of probability of transformer being in bad, medium or good conditions are shown on figure 5.

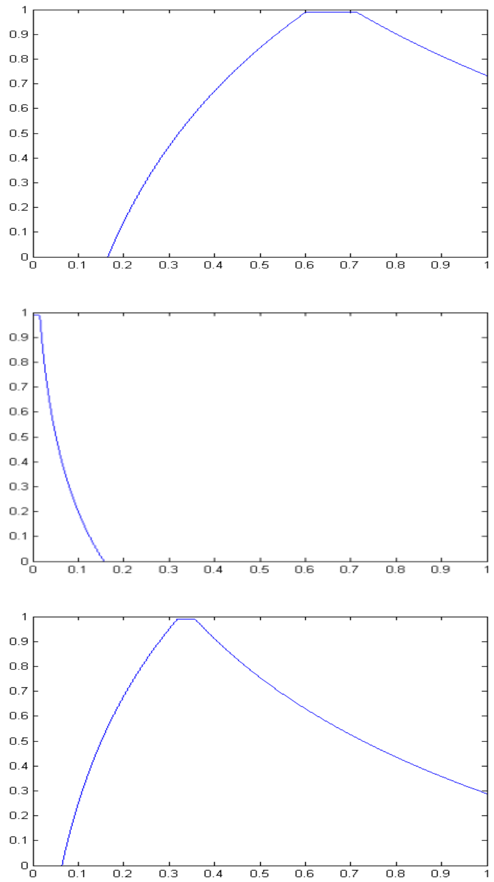

Figure 5. Fuzzy values of probability of transformer being in a) bad, b) medium and c) good condition.

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V. CONCLUSION

The utilization of a probability measure for uncertainty modeling requires too much precise information in the form of prior and conditional probability tables, and such information is often difficult or impossible to obtain. In this paper, a verbal expression or interval value of probabilistic uncertainty is proved to be more appropriate than numerical values. The fuzzy influence diagram with the fuzzified probabilities of states is presented in the paper.

Calculation of these probabilities is performed with interval based fuzzy arithmetic. Results presented in case studies proved that this new form of description - fuzzy influence diagram, that is both a formal description of the problem that can be treated by computers and a simple, easily understood representation of the problem can be successfully implemented for various class of risk analysis problems in power systems.

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