Control of a Quadrotor Flight

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Abstract – *This paper presents the control of a quadrotor flight. Two characteristic representatives of frequently used flight control techniques are considered: backstepping and fuzzy. The paper aims to contribute to the objective assessment of quadrotor control performances with respect to the criteria regarding to dynamic performances, trajectory tracking precision, energy efficiency and control robustness upon stochastic internal and/or external perturbation. Qualitative evaluation of the closed-loop system performance should to enable the best choice of quadrotor control structure. Non-linear modeling, control and numerical simulation of characteristic flight test-scenarios are described in the paper, too. Obtained simulation results for two representative control algorithms are graphically and table presented, analyzed and discussed.*

1. INTRODUCTION

Quadrotor helicopters increasingly attract the attention of potential researchers. The quadrotor architecture has been chosen for this research for its low dimension, good maneuverability, simple mechanics and payload capability. This paper is addressed to problems of controller performances evaluation and analysis. The main benefits of this research concern with achievement of a controller architecture that should enable quadrotor high dynamic performances, robustness to external perturbations as well as satisfactory trajectory tracking precision.

The linear control techniques based on PID feedback structure [1-2] are frequently used with micro copters for flight control. The strength of the PID feedback is the exponential convergence property mainly due to the compensation of the Coriolis and gyroscopic terms. On the contrary a PID structure does not require some specific model parameters and the control law is much simpler to implement.

In the recent time, very popular control technique is done with backstepping control [3-4]. In the respective publications the convergence of the quadrotor internal states is guaranteed, but a lot of computation is required.

Other control algorithms belong to the class so called the knowledge-based algorithms. Main characteristics of these methods are that they represent non-linear techniques that do not require knowledge about the model of system. These techniques assume quadrotor plant as a black-box. They use control platform with fuzzy techniques [5].

The paper is organized as follows: Section 1:Introduction. In Section 2, the quadrotor dynamics modeling is presented. In Section 3 modeling of the control strategy is presented. In Section 4, the simulation experiments and flight controller evaluation are illustrated. Conclusions are given in Section 5.

2. QUADROTOR DYNAMICS MODELING

The model of the quadrotor helicopter and the rotational directions of the propellers can be see in Figure 1. This cross structure is quite thin and light, however it shows robustness by linking mechanically the motors (which are heavier than the structure). Each propeller is connected to the motor through the reduction gears. All the propellers axes of rotation are fixed and parallel. These considerations point out that the structure is quite rigid and the only things that can vary are the propeller speeds.

As shown in Fig. 1.1, one pair of opposite propellers of quadrotor rotates clock-wise (2 and 4), whereas the other pair rotates anticlockwise (1 and 3). This way it is able to avoid the yaw drift due to reactive torques. This configuration also offers the advantage of lateral motion without changing the pitch of the propeller blades. Fixed pitch simplifies rotor mechanics and reduces the gyroscopic effects. Control of quadrotor is achieved by commanding different speeds to different propellers, which in turn produces differential aerodynamic forces and moments. For hovering, all four propellers rotate at same speed. For vertical motion, the speed of all four propellers is increased or decreased by the same amount, simultaneously. In order to pitch and move laterally in that direction, speed of propellers 3 and 1 is changed conversely. Similarly, for roll and corresponding lateral motion, speed of propellers 2 and 4 is changed conversely. To produce yaw, the speed of one pair of two oppositely placed propellers is increased while the speed of the other pair is decreased by the same amount. This way, overall thrust produced is same, but dif-ferential drag moment creates yawing motion. In spite of four actuators, the quadrotor is still an under-actuated system.

The Figure 1. shows the structure model [6-9] in hovering condition, where all the propellers have the same speed of rotation $\mu_i = \mu_H, i = 1,...,4$. In the Fig. 1. all the propellers rotate at the same (hovering) speed μ_H (rad/s) to counterbalance the acceleration due to gravity. Thus, the quadrotor performs stationary flight and no forces or torques moves it from its position. Even though, the quadrotor has 6 DOFs, it is equipped just with four propellers hence it is not pos-sible to reach a desired set-

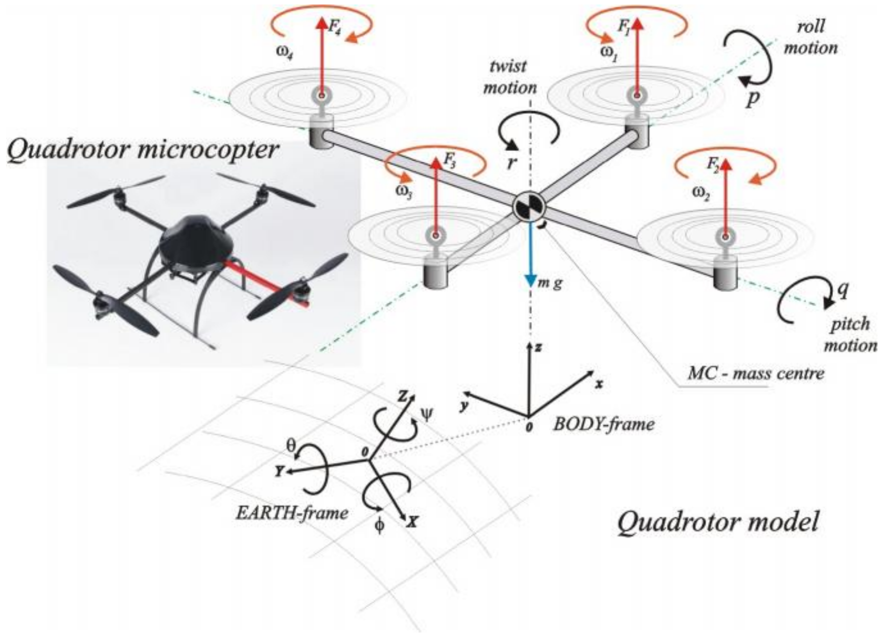

Figure 1. Quadrotor rotorcraft – a non-linear dynamic system, coordinate systems assumed to enable model derivation.

point for all the DOFs, but at maximum four. However, thanks to its structure, it is quite easy to choose the four best controllable variables and to decouple them to make the controller easier. The four quadrotor targets are thus related to the four basic movements which allow the microcopter to reach a certain height and attitude.

Dynamic modelling of the quadrotor helicopter is a well elaborated field of aeronautics. To describe the motion of a 6 DOF rigid body it is usual to define two reference frames (Fig. 1):

(i) the earth inertial frame (E-frame), and
(ii) the body-fixed frame (B-frame).

The equations of motion are more conveniently formulated in the B-frame because of the following reasons:

(i) the inertia matrix is time-invariant;
(ii) advantage of body symmetry can be taken to simplify the equations;
(iii) measurements taken on-board are easily converted to body-fixed frame; Control forces are almost always given in body-fixed frame.

The linear position of the helicopter (X,Y,Z) is determined by the coordinates of the vector between the origin of the B-frame and the origin of the E-frame accord-ing to equation. The angular position (or attitude) of the helicopter (¤, ¯, ·) is defined by the orientation of the B-frame with respect to the E-frame. The vector that describes quadrotor position and orientation is:

$$s = [X \quad Y \quad Z \quad ¤ \quad ¯ \quad ·]^T \qquad (1)$$

The generalized quadrotor velocity expressed in the B-frame can be written as:

$$v = [u \quad ³ \quad w \quad p \quad q \quad r]^T \qquad (2)$$

where, the $u, ³, w$ represent linear velocity components in the B-frame, while p, q, r are corresponding angular velocities of rotation about corresponding roll, pitch and yaw axes. Finally, the kinematical model of the quadrotor correlates the motions in these two coordinate systems.

The quadrotor dynamics can be described in the known form, extended by adding of the air-resistance member. The equation that describes model in B-frame is:

$$M_B \dot{v} + C_B(v)v + G_B(s) + R_a(v) = O_B(v)\Omega + E_B\Omega^2 \quad (3)$$

where:
$\dot{v}$ is the generalized acceleration vector with respect to (w.r.t.) the B-frame,
M_B is the system inertia matrix,
C_B is the Coriolis-centripetal matrix and
G_B is the gravitational force vector, all expressed w.r.t. the B-frame.
The R_a is the air-resistance vector.
The O_B and E_B are the gyroscopic propeller matrix and the movement matrix, successively.

The gyroscopic propeller matrix O_B depends on total rotational moment of inertia around the propeller axis and corresponding angular speeds p, q. The matrix E_B depends on the design parameters – thrust and drag coefficients. Air resistance forces appear as an external perturbation to the quadrotor translational movements in longitudinal (x), lateral (y) and normal (z) direction w.r.t B-frame. The drag force R_a depends on a magnitude of body-fluid relative velocity. It takes into account both - air resistance and wind gust.

Equation (3), after certain rearrangement and transformation from the B-frame space to E-frame space, can be written in the scalar form suitable for controller design. Now, the model of quadrotor dynamics can be described by a system of equations:

$$\ddot{X} = (\sin \cdot \ \sin \text{¤} + \cos \cdot \ \sin^{-} \cos \text{¤})\frac{U_1}{m} - \frac{R_{a,x}}{m}$$

$$\ddot{Y} = (-\cos \cdot \ \sin \text{¤} + \sin \cdot \ \sin^{-} \cos \text{¤})\frac{U_1}{m} - \frac{R_{a,y}}{m}$$

$$\ddot{Z} = -g + \cos^{-} \cos \text{¤} \frac{U_1}{m} - \frac{R_{a,z}}{m}$$

$$\ddot{\text{¤}} = \frac{I_{YY} - I_{ZZ}}{I_{XX}} \dot{-}\ \dot{\cdot}\ r - \frac{J_{TP}}{I_{XX}} \dot{-}\ \Omega_r + \frac{U_2}{I_{XX}} \tag{4}$$

$$\ddot{-} = \frac{I_{ZZ} - I_{XX}}{I_{YY}} \dot{\text{¤}}\ \dot{\cdot} + \frac{J_{TP}}{I_{YY}} \dot{\text{¤}}\ \Omega_r + \frac{U_3}{I_{YY}}$$

$$\ddot{\cdot} = \frac{I_{XX} - I_{YY}}{I_{ZZ}} \dot{\text{¤}}\ \dot{-} + \frac{U_4}{I_{ZZ}}$$

The overall propeller's speed (rad/s) is defined by equation (11):

$$\Omega_r = -\mu_1 + \mu_2 - \mu_3 + \mu_4 \tag{5}$$

Quadrotor is equipped with four fixed-pitch rotors, each one includes a Brash-Less Direct Current (BLDC) motor, a one-stage gearbox and a rotary-wing (propeller).

3. MODELING OF THE CONTROL STRATEGY

Commonly used control system architecture of autonomous quadrotor is presented in Fig. 2. The task planning block is in debt to determine referent 3D rotorcraft trajectory as well as to propose the referent flight speed along the trajectory. The task planning block generates referent path based on flight parameters and microcopter task imposed.

Position control block has to ensure accurate 3D trajectory tracking. It represents so called outside control loop. Based on sensory information (GPS, IR, SONAR) about the referent positions (speeds) and corresponding actual ones defined in the inertial coordinate system (E-frame), the position controller calculates referent attitude position of quadrotor body (pitch $^{-}_{ref}$ and roll angle ¤_{ref}) that have to enable desired motion.

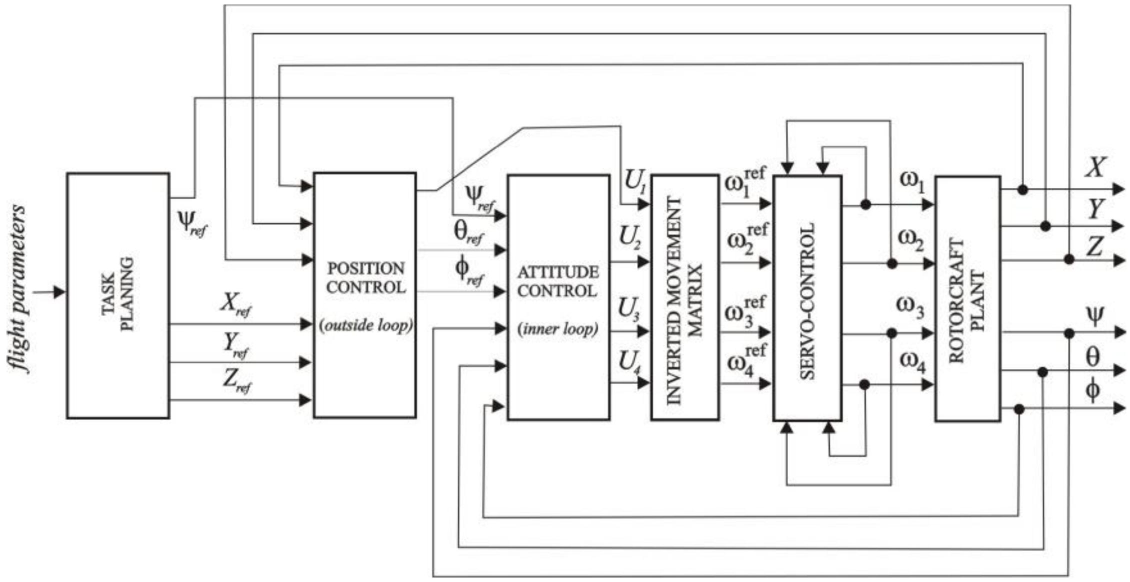

Figure 2. Block-scheme of the global control system architecture of autonomous quadrotor.

Inner control block represents the core of the control scheme. It is responsible for the attitude control of quadrotor system. Appropriate attitude control ensures in an indirect way required flight performances in the particular directions of motion such as longitudinal, lateral as well as vertical. Inner control block processes the task and sensor data and provides a signal for basic movements which balances the position error. Equation (4) is used in this bock to transfer an acceleration command to a basic movement one.

The control rules to be used to estimate the acceleration commands are to be considered in the next section.

The essence of building control scheme presented in Fig. 2 is that by control-ling a body attitude (within an inner loop) it is enabled controlling of the linear rotorcraft movements. Also, high robustness to parameter and structural uncertainties of system modeling are required in design of attitude control algorithm.

Inverted Movements Matrix block (Fig. 2) is used to compute the propeller's squared speed from the four basic movement signals.

Variety of control algorithms can be implemented within the flight controller presented in Fig. 2. This paper aims to propose corresponding testing procedure and a qualitative evaluation of three representative flight control techniques. These are:

(i) non-linear, model-based backstepping method (BSM), and
(ii) non-linear, knowledge-based fuzzy logic control (FLC) based on use of a Fuzzy Inference System.

3.1 Backstepping Controller

The backstepping technique is recursive design methodology that makes use of Lyapunov stability theory to force the system to follow a desired trajectory. Backstepping approach to quadrotor flight control was successfully applied in number of researches [3-4]. First, the dynamical model is rewritten in state-space form $\dot{X} = f(X,U)$, by introducing $X = [x_1..x_{12}]^T \in \Re^{12}$ as space vector of the system:

$$\begin{aligned} x_1 &= \phi & x_5 &= \psi & x_9 &= Y \\ x_2 &= \dot{x}_1 = \dot{\phi} & x_6 &= \dot{x}_5 = \dot{\psi} & x_{10} &= \dot{x}_9 = \dot{Y} \\ x_3 &= \theta & x_7 &= X & x_{11} &= Z \\ x_4 &= \dot{x}_3 = \dot{\theta} & x_8 &= \dot{x}_7 = \dot{X} & x_{12} &= \dot{x}_{11} = \dot{Z} \end{aligned} \tag{6}$$

Next, the x- coordinates are transformed into the new z - coordinates by means of a diffeomorphism:

$$\begin{aligned} z_1 &= x_{1\_ref} - x_1 & z_7 &= x_{7\_ref} - x_7 \\ z_2 &= x_2 - \dot{x}_{1\_ref} - \alpha_1 z_1 & z_8 &= x_8 - \dot{x}_{7\_ref} - \alpha_7 z_7 \\ z_3 &= x_{3\_ref} - x_3 & z_9 &= x_{9\_ref} - x_9 \\ z_4 &= x_4 - \dot{x}_{3\_ref} - \alpha_3 z_3 & z_{10} &= x_{10} - \dot{x}_{9\_ref} - \alpha_9 z_9 \\ z_5 &= x_{5\_ref} - x_5 & z_{11} &= x_{11\_ref} - x_{11} \\ z_6 &= x_6 - \dot{x}_{5\_ref} - \alpha_5 z_5 & z_{12} &= x_{12} - \dot{x}_{11\_ref} - \alpha_{11} z_{11} \end{aligned} \tag{7}$$

By introducing the partial Lyapunov functions to all x - coordinates results in the following backstepping controller:

$$\begin{aligned} U_X &= \frac{m}{U_1}(z_7 - \alpha_7(z_8 + \alpha_7 z_7) - \alpha_8 z_8) \\ U_Y &= \frac{m}{U_1}(z_9 - \alpha_9(z_{10} + \alpha_9 z_9) - \alpha_{10} z_{10}) \\ U_1 &= \frac{m}{\cos x_1 \cos x_3}(z_{11} + g - \alpha_{11}(z_{12} + \alpha_{11} z_{11}) - \alpha_{12} z_{12}) \\ U_2 &= I_{XX}(z_1 - \frac{I_{YY} - I_{ZZ}}{I_{XX}} x_4 x_6 + \frac{J_{TP}}{I_{XX}} x_4 \Omega_r - \alpha_1(z_2 + \alpha_1 z_1) - \alpha_2 z_2) \\ U_3 &= I_{YY}(z_3 - \frac{I_{ZZ} - I_{XX}}{I_{YY}} x_2 x_6 - \frac{J_{TP}}{I_{XX}} x_2 \Omega_r - \alpha_3(z_4 + \alpha_3 z_3) - \alpha_4 z_4) \\ U_4 &= I_{ZZ}(z_5 - \frac{I_{XX} - I_{YY}}{I_{ZZ}} x_2 x_4 - \alpha_5(z_6 + \alpha_5 z_5) - \alpha_6 z_6) \end{aligned} \tag{8}$$

3.2 Fuzzy Controller

Fuzzy controllers can be designed intuitively in light of the knowledge acquired on the behavior of the system. This knowledge is often gained through experience and common sense, regardless of the mathematical model of the dynamics governing its behavior, and it is in the form of set of rules. The controller that will be implemented here consists of six FLCs, one for each particular state, that are in form of zero order TSK fuzzy inference system. FLC described in this paper with error e and the error rate $\dot{e}$. Actually, inputs in the FLC are first preprocessed, then they are normalized to fit membership function intervals [-1, 1] and [-3, 3], and finally feed to FLC. The output of the FLC is control action u. Each input variable possess the corresponding three fuzzy sets NEGATIVE, ZERO and POSITIVE and they are presented in Fig. 3. Output membership functions are fuzzy singletons.

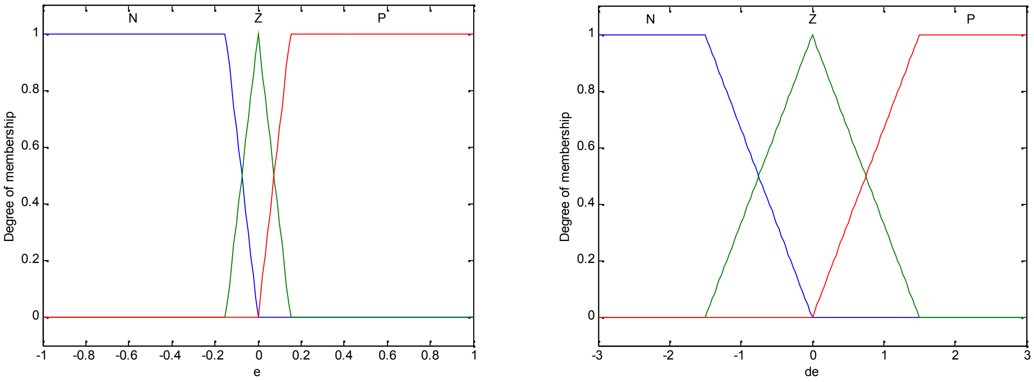

Figure 3. Membership functions for input variables

Fuzzy rule base			
Rule No	Input e	Input $\dot{e}$	Output u
1	NEGATIVE	NEGATIVE	NEGATIVE
2	NEGATIVE	ZERO	NEGATIVE
3	NEGATIVE	POSITIVE	ZERO
4	ZERO	NEGATIVE	NEGATIVE
5	ZERO	ZERO	ZERO
6	ZERO	POSITIVE	POSITIVE
7	POSITIVE	NEGATIVE	ZERO
8	POSITIVE	ZERO	POSITIVE
9	POSITIVE	POSITIVE	POSITIVE

Table 1. Table of fuzzy rules used for the flight control of quadrotor

4. SIMULATION EXPERIMENTS AND FLIGHT CONTROLLER EVALUATION

For the purpose of testing flight controller the following quadrotor parameters are assumed (Tab.2).

Model parameters					
Symbol	Value	Unit	Symbol	Value	Unit
m	1.0	kg	o	1.225	kg/m^3
I_{XX}, I_{YY}	$8.1 \cdot 10^{-3}$	Nms^2	A_x, A_y	0.0121	m^2
I_{ZZ}	$14.2 \cdot 10^{-3}$	Nms^2	A_z	0.0143	m^2
b	$54.2 \cdot 10^{-6}$	Ns^2	$C_{d,x}, C_{d,y}$	1.125	-
d	$1.1 \cdot 10^{-6}$	Nms^2	$C_{d,z}$	1.04	-
l	0.24	m	K_m	0.973	-
J_{TP}	$104 \cdot 10^{-6}$	-	T_m	0.113	-
g	9.81	m/s^2	2_m	0.0012	s

Table 2. Quadrotor model parameters used in simulation experiments

Control parameters from Tab. 2 are determined by simulation. For the purpose of analysis and qualitative evaluation of quadrotor flight controller performances, two representative control algorithms (BSM and FLC) are considered. Fuzzy control parameters are given in Figure 3 and Tab. 1. Control parameters of the Backstepping controller are given in Tab. 3.

$\sim_1$	$\sim_2$	$\sim_3$	$\sim_4$	$\sim_5$	$\sim_6$	$\sim_7$	$\sim_8$	$\sim_9$	$\sim_{10}$	$\sim_{11}$	$\sim_{12}$
10.7	2.0	9.5	3.8	2.2	2.1	2.0	3.0	2.0	3.0	3.0	3.0

Table 3. Backstepping controller parameters used in simulation

Assessment and qualitative evaluation of two representative control techniques frequently used with UMAV are accomplished upon the criteria imposed. The following criteria are introduced: (i) criterion on fine dynamic performances; (ii) criterion on trajectory tracking accuracy; (iii) criterion on control robustness upon the external perturbation; and (iv)criterion on energy efficiency. Control algorithms chosen are evaluated by comparison of the simulation results obtained for the same control object and same flight conditions. Dynamic quadrotor flight in the 3D-loop manoeuvre experimental scenarios is considered as the characteristic benchmarking procedures. Chosen benchmarking tests enable credible assessment of different control techniques under the same conditions. Dynamic quadrotor flight regards to a microcopter movement in the perpendicular planes in a rather narrow space (Fig. 4). It is accomplished by flying in the 3D-loop about a horizontal and a vertical rod set in such a way to be 2 meters far one from another. The curve-linear, smooth loop (trajectory) is predefined by introducing 8 key-waypoints (Fig. 4). The trajectory defined includes several flight maneuvers: (i) throttle movements in the vertical direction (1-2 and 7-8), (ii) counter-clockwise roll movements (2-3-4 and 5-6-7), (iii) tilt movement about the pitch axis (4-5), and short (iv) hovering with the constant propeller speed (in the point 2 i.e. 7). Quadrotor is required to track the imposed trajectory-loop shown in Fig. 4 moving along at a low speed of maximal value 0.5 (m/s) and to repeat the same path for 33% increased average speed with maximum of 1 (m/s). Flying in the loop, quadrotor is subjected to influence of the inertia and centripetal forces that tend to run a rotorcraft away from the desired path as well as to disturb its dynamic performances (keeping attitude within the allowed range, smooth acceleration profile, no vibration and turbulence).

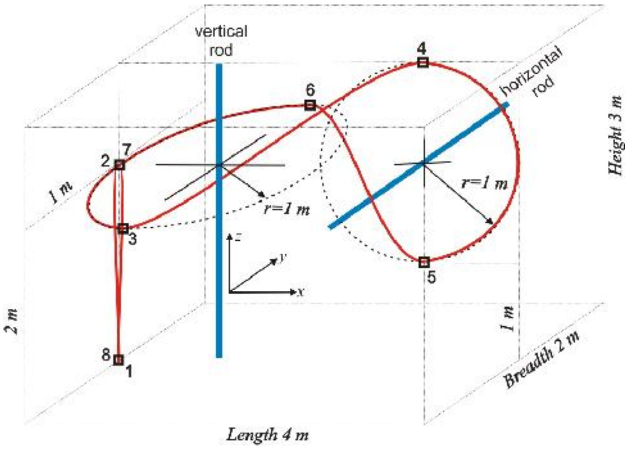

Figure 4. Trajectory-loop for testing of quadrotor dynamic flight perfromances

By analysis of the simulation results, Backstepping method ensures the best control performances in sense of trajectory tracking precision. FLC controller have slightly better characteristics in sense of energy efficiency (less consumptions). By increasing of flight speed dynamic effects become influential upon the system performances. Consequently, Backstepping method is more sensitive to changing of flight speed than FLC. Degradation of control system performances with excitation of dynamic modes in the case of BSM implementation are shown in Figs. 5 and 6.

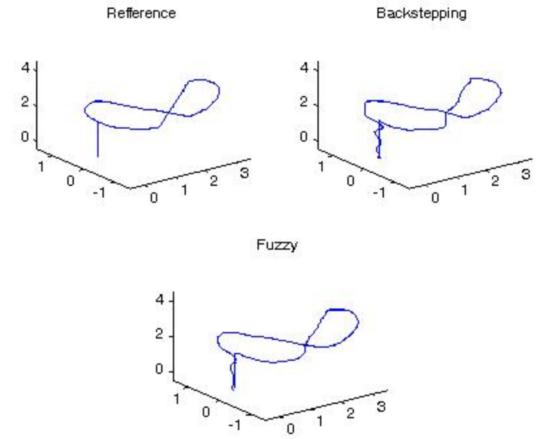

Figure 5. Trajectory tracking accuracy of the reference path obtained for three examined control techniques and for the case of low speed flight

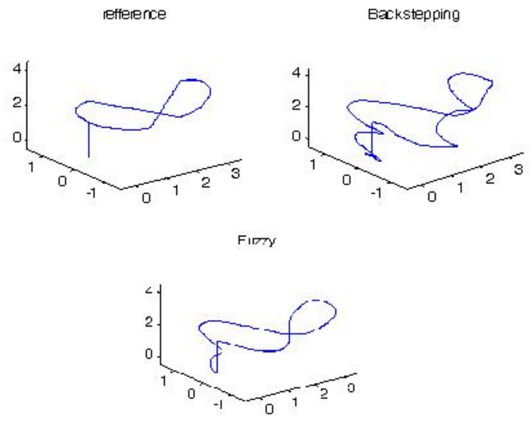

Figure 6. Trajectory tracking accuracy of the reference path obtained for three considered control techniques and for the case of increased flight speed

5. CONCLUSIONS

The paper regards to development of appropriate benchmarking and qualitative evaluation procedures dedicated to exploration, analysis and validation of flight controller performances of quadrotor UMAVs. The credible benchmark simulation test is proposed in the paper. The indoor test, capable for exploration of dynamic flight scenarios is simulated. Two controllers (Backstepping and Fuzzy controller) as typical representatives of non-linear and model-based knowledge-based control techniques are validated through several closed-loop simulation tests. Based on a qualitative analysis of the obtained simulation results the Backstepping controller was identified as the best flight controller solution. Proposed benchmark and evaluation procedure, described in the paper, can be usefully implemented in evaluation of other control methods in the same way, too.

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