Performance analysis of FSO systems based on a new shadowed Chi-square PDF scintillation model

Bojana Milosavljevic,[1[0000-0002-8322-7598]] Stefan Panic,[2[0000-0002-5868-1764]] Srdjan Milosavljevic,[3[0000-0003-2199-0632]] and Petar Spalevic[4[0000-0002-6867-7259]]

[1] Kosovo and Metohija Academy of applied studies, Serbia
[2] Faculty of Natural Science and Mathematics, University of Pristina, Serbia
[3] Faculty of Economics, University of Pristina, Serbia
[4] Faculty of Technical Science, University of Pristina
bojana.milosavljevic@pr.ac.rs

Abstract. In this paper, we will present novel general model for the irradiance FSO fluctuations, based on the Chi-square distribution. The error performance of the Free Space Optical (FSO) system modulated with On-Off keying (OOK) scheme both in the presence of atmospheric turbulence and misalignment fading (i.e. pointing error) will be investigated. For both cases, the expressions for the Average Bit Error Rate (ABER) will be determined in analytically closed form and discussed in function of average optical power at the transmission. The results will be graphically presented in order to determine impact of relevant parameters on the quality of the received signal in the OOK modulated FSO system.

Keywords: Free Space Optics (FSO). Turbulence. Pointing error, ABER (Average Bit error rate).

1 Introduction

Free Space Optical (FSO) represent Line-of-Sight (LOS) optical transmission techniques that are used to establish communication over distances up to several kilometers through the atmosphere offering high bandwidth capacity. They provide higher level of security and they are more resistant to interference then radiofrequency (RF) systems. Such links are suitable for a bandwidth of several Gb/s. FSO systems are very vulnerable to atmospheric conditions. Since signal propagation is done in free space, it is affected by atmospheric turbulence and pointing errors, which deteriorate the system performance.

One of the main issues in the performance analysis of Free-Space Optical (FSO) communication is the introduction of an analytically tractable Probability Density Function (PDF) model for the random amplitude fluctuation of the signal due to atmospheric turbulence. Various FSO channel models have been proposed in the literature to model the impact of different turbulence levels, such as the Lognormal-Rician model [1], K-model [2], I-K model [3], Gamma-Gamma model [4], Malaga model [5], dual Weibull model [6], and dual generalized Gamma model [4]. All of these models provide results that match measurements for a wide range of turbulence conditions, as they consider the effects of large-scale and small-scale scintillations caused by vortex turbulence. In [7], [8], the Longnormal-Rician distribution was shown to align with simulation data based on experimental measurements. However, in [8], the expressions for the PDF and CDF of the Lognormal-Rician model are approximated due to their inability to be analytically expressed in closed forms. Rician (Chi-square) model has been presented in [9]-[14] as a highly efficient model for modeling a broad spectrum of vortex turbulence (eddy effect). In line with the aforementioned, an analytical Gamma-Chi-square (Gamma-Rician) PDF composite model has been proposed, which combines the Gamma model and Chi-square (Rician) model. Based on the analyzed results from [8], [9], [15], [16], [17], it is concluded that the Gamma model is reliable for analyzing small-scale scintillations under the influence of vortex turbulence, while the Chi-square (Rician) model is efficient in describing the effects of large-scale scintillations caused by vortex turbulence. Furthermore, this PDF model is generalized to account for the performance degradation effects due to induced fading mismatch by introducing an error model of misalignment into the obtained Gamma-Chi-square distribution, similar to [18]-[21]. In order to perform a comprehensive performance analysis of FSO communication using On-Off Keying (OOK) modulation format for the introduced channel model, we will consider the average bit error rate (ABER) values and analyze them as functions of the parameters of the transmit power system. For both cases, in the presence of atmospheric turbulence with and without pointing errors, the ABER expressions at the receiver side will be presented in an analytically closed form. The analytically obtained results will be verified with the results obtained from Monte Carlo simulations.

2 Atmosphere turbulence model

We have already mentioned the well-known approach in scintillation theory [15], where the received optical wave radiation intensity is obtained as the product of radiation intensities occurring in the channel due to the effects of large-scale and small-scale scintillations, i.e. $I_a = I_x I_y$, assuming that I_x and I_y are statistically independent random processes. Generally, wireless optical signal is affected by refractive index variation, which leads to the interaction between the optical signal and vortex turbulence, resulting in phase and amplitude fluctuations of the received signal [5], [15], [22]. In line with the theory, we assume that both small-scale and large-scale fluctuations act simultaneously in the Gamma-Chi-square channel. The Chi-square model of the amplitude distribution of the optical signal is given by the following expression [9], [23]:

$$f_{I_x}\left(I_x\right) = \frac{1+K}{\Omega_P} \mathrm{e}^{-K-\frac{(1+K)I_x}{\Omega_P}} I_0\left(2\sqrt{\frac{K\left(1+K\right)}{\Omega_P} I_x}\right), \quad I_x > 0 \tag{1}$$

Where I_x represents the amplitude of the optical signal, parameter K defines the power ratio between the line-of-sight (LOS) component and the scattered radiation component [9], Ω_r denotes the total received signal power, and I_υ (*) is the modified Bessel function of the first kind of order υ [24, Eq.8.431].

In this section, we will focus on modeling the effects of small-scale scintillations caused by vortex turbulence, I_y, by modeling this random process using the probability density function of turbulence proposed in [32]:

$$f_{I_y}\left(I_y\right)=\left(\frac{g\beta}{g\beta+\Omega'}\right)^{\beta}\frac{1}{g}e^{-\frac{I_y}{g}}{}_1F_1\left(\beta,1,\frac{\Omega' I_y}{g\left(g\beta+\Omega'\right)}\right),\quad I_y>0 \tag{2}$$

where parameter β represents the turbulence intensity in the channel, parameter g, g=2b_0(1-ρ), represents the total power of scattered signal components, parameter b_0 represents the average power of scattered signal components, and parameter ρ, (0 ≤ ρ ≤ 1), represents the power of scattered signal components coupled with the LOS (Line-of-sight) signal component. Parameter **Ω'**, **Ω'** = **Ω** + 2b_0ρ + 2√2b_0ρ**Ω** cos(φ_A-φ_B), represents the average power of the optical signal, where parameter **Ω** represents the contribution of the average power of the LOS signal component, and φ_A and φ_B represent the deterministic phases of the LOS signal component and the component of the signal coupled with the LOS component.
Using the relation $I_a = I_x I_y$, the PDF for the atmospheric turbulence model can be obtained as:

$$f_{I_a}\left(I_a\right)=\int_0^{\infty} f_{I_x}\left(I_a\middle|I_y\right)f_{I_y}\left(I_y\right)dI_y \tag{3}$$

by substituting equations (1) and (2) into equation (3) and representing the modified Bessel function of the first kind I_υ (*) in equation (1) as an infinite series [24, Eq.8.445], and using the solution of the resulting integral given in [24, Eq.3.478.4], we obtain a closed-form expression for the PDF model of the Gamma-Chi-square distribution in the form [27]:

$$f_{I_a}\left(I_a\right)=\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{2\left(1+K\right)^{\frac{p+q}{2}+1}K^p\,\mathrm{e}^{-K}\,g^{\beta-\frac{p+q}{2}-1}\beta^{\beta}\left(\Omega'\right)^q\left(\beta\right)_q}{\Gamma\left(p+1\right)p!\left(q!\right)^2\Omega_P^{\frac{p+q}{2}+1}\left(g\beta+\Omega'\right)^{\beta+q}}I_a^{\frac{q+p}{2}}K_{(q-p)}\left(2\sqrt{\frac{\left(1+K\right)}{\Omega_P g}I_a}\right) \tag{4}$$

where K_υ(*) represents the modified Bessel function of the second kind of order υ [24, Eq.8.432].

This result represents a significant contribution because such a distribution model of turbulence in the channel has not been considered in the literature before, and it represents a general model that can be reduced to previously known distribution models by setting appropriate parameter values. Figures 1, and 2 represent the PDF values of the signal intensity transmitted through the shaded Chi-square model of the optical channel observed for different system parameter values.

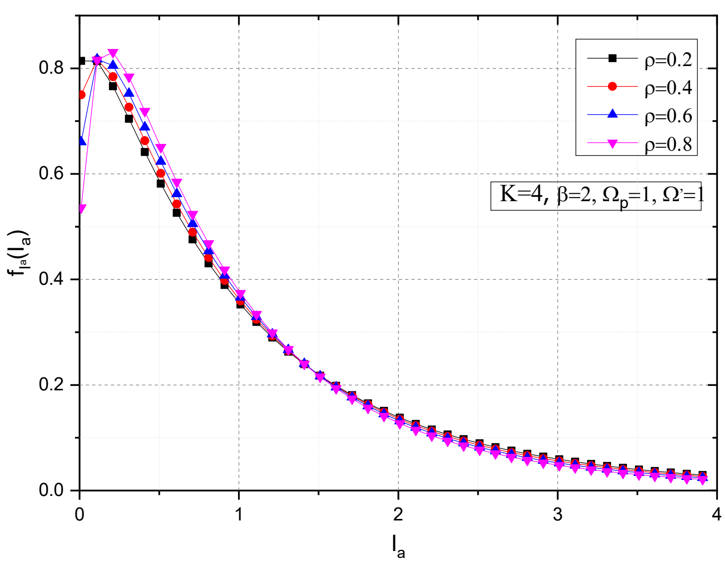

Fig. 1. PDF for the shadowed Chi-square model of the optical channel observed for different cases of coupling between the scattered components and the LOS component of the signal.

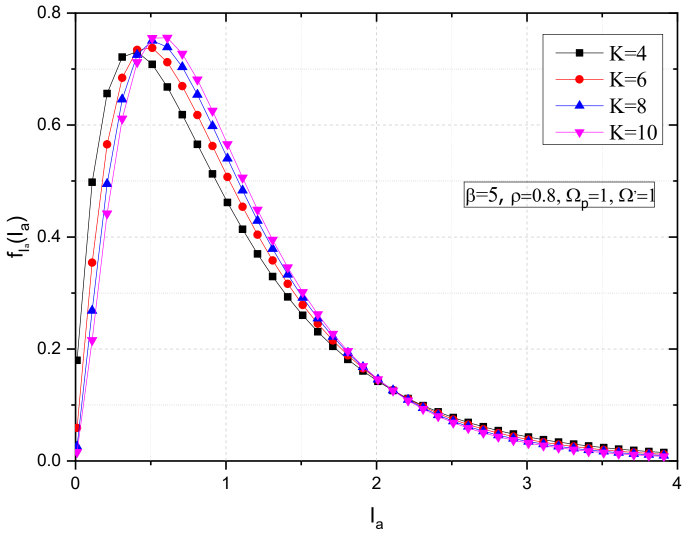

Fig. 2. PDF for the shadowed Chi-square model of the optical channel observed for different cases of varying signal intensities.

3 Pointing error model

In addition to atmospheric turbulence, positioning error is another factor that can affect signal attenuation in FSO systems, specifically due to misalignment between the transmitting laser and the receiving detector. The Non-zero Boresight model PDF of the optical signal describes the positioning error, which is considered to include the radius of the optical beam at a certain distance from the transmitter, the radius of the circular detector aperture, and the variance of misalignment. It is presented in the form [8], [20], [25], [26]:

$$f_{I_p}\left(I_p\right)=\frac{\xi^2}{A_0^{\xi^2}}I_p^{\xi^2-1}, \quad 0\le I_p\le A_0 \tag{5}$$

parameter $A_0 = erf(v)^2$ represents the maximum fraction of directed power, where $erf(*)$ is the error function [24, Eq.8.250], parameter $v = \sqrt{(\pi\, a/(\sqrt{2}\, w_L))}$ and a represents the radius of the circular detector aperture, while w_L is the radius of the optical beam at distance L from the transmitter. Parameter $\xi = w_{Leq}/(2\sigma_s)$ represents the ratio of the equivalent beam radius at the receiver w_{Leq} and the standard deviation of the receiver's jitter σ_s, where $w^2_{Leq} = w_L^2 \sqrt{(\pi\, erf(v)/[2v \exp(-v^2)]}$.

Using the same procedure as in [18], and using the relation $I = I_a I_p$ for this positioning error model, the new PDF can be obtained based on:

$$f_I(I) = \int_{I/A_0}^{\infty} f_{I_p}(I|I_a) f_{I_a}(I_a) dI_a \tag{6}$$

By substituting equations (4) and (5) into equation (6) and expressing the modified Bessel function of the second kind Kυ(*) through the Meijer G function as in [24, Eq.9.34.3], and using the solution of the resulting integral given in [28, Eq.07.34.21.0082.01], we obtain the expression for the PDF of the Gamma-Chi-square model with positioning error in the form [27]:

$$f_I(I) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{\xi^2 (1+K)^{\frac{p+q}{2}+1} K^p \mathrm{e}^{-K} g^{\beta-\frac{p+q}{2}-1} (\Omega')^q \beta^\beta (\beta)_q A_0^{\frac{p+q}{2}+2}}{\Gamma(p+1) p!(q!)^2 \Omega_P^{\frac{p+q}{2}+1} (g\beta+\Omega')^{\beta+q}} I^{\frac{p+q}{2}+1} G_{1,3}^{3,0}\left[\frac{(1+K)}{A_0 \Omega_P g} I \middle| \begin{matrix} \xi^2 - 1 - \frac{p+q}{2} \\ \xi^2 - 1 - \frac{p+q}{2}, \frac{q-p}{2}, \frac{p-q}{2} \end{matrix}\right] \tag{7}$$

The result (7) also represents a significant contribution of the paper as it presents a general model for the distribution of the optical channel amplitude in the presence of turbulence and positioning effects.

4 Performance analysis

The average bit error rate (ABER) as a function of SNR for an optical signal transmitted through an FSO system with OOK modulation scheme can be calculated based on [29], [30]:

$$P_e = \int_0^{\infty} \frac{1}{2} \mathrm{erfc}\left(\frac{\sqrt{\gamma}}{2}\right) f_\gamma(\gamma) d\gamma \tag{8}$$

where $f_\gamma(\gamma)$ represents the PDF for the current SNR of the received signal.

By substituting equations (4) into equation (8), and then representing erfc() using the Meijer G function as in [31, Eq.06.27.26.0003.01], and the modified Bessel function of the second kind Kυ-(*) through the Meijer G function as in [24, Eq.9.34.3], we obtain the ABER expression for atmospheric turbulence Gamma-Chi-square channel model as a function of average optical power transmission P_T, in the following form:

$$P_e(P_T) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{\xi^2 K^p \mathrm{e}^{-K} g^{\beta+1} \beta^\beta (\Omega')^q (\beta)_q \Omega_P}{4\sqrt{\pi}\Gamma(p+1) p!(q!)^2 (g\beta+\Omega')^{\beta+q}} H_{2,5}^{5,1}\left[\left(\frac{P_T \Omega_P g A_0}{\sigma(1+K)}\right)^2 \middle| \begin{matrix} (1,1), \ (-\xi,2) \\ (0,1), \ \left(\frac{1}{2},1\right), \ (-\xi,2), \ \left(-\frac{3p}{2}-\frac{q}{2},2\right), \ \left(-\frac{3q}{2}-\frac{p}{2},2\right) \end{matrix}\right] \tag{9}$$

where:

$$H_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_1,A_1),\ldots,(a_p,A_p)\\(b_1,B_1),\ldots,(b_q,B_q)\end{matrix}\right.\right]$$

represents Fox H function.

5 Numerical results

For the numerical calculation, the FSO system was observed at the wavelength λ = 1550 nm and at the distance between transmitter and receiver L = 1 km. Three types of atmospheric turbulence were considered: weak, moderate and strong, with indexes of reflection $C_n^2 = 6\cdot10^{-15}$ $m^{-2/3}$, $C_n^2 = 2\cdot10^{-14}$ $m^{-2/3}$ and $C_n^2 = 1.2\cdot10^{-13}$ $m^{-2/3}$, respectively. Total received signal power is $\Omega_P = 1$, while detector responsivity R = 1 A/W and noise variance is $\sigma_N = 10^{-7}$ A/Hz. The radius of a circular detector aperture $a = 0.05$ m, optical beam radius at distance L from transmitter $w_L = 0.5$ m, pointing error displacement standard deviation (jitter) at the receiver $\sigma_s = 0.2$ m are considered.
In Figs. 3 and 4 is shown the ABER behavior of the FSO channel modeled with the Shadowed Chi-square/Chi-square distribution, as a function of average electrical SNR in the case of atmospheric turbulence and a pointing error model, respectively.

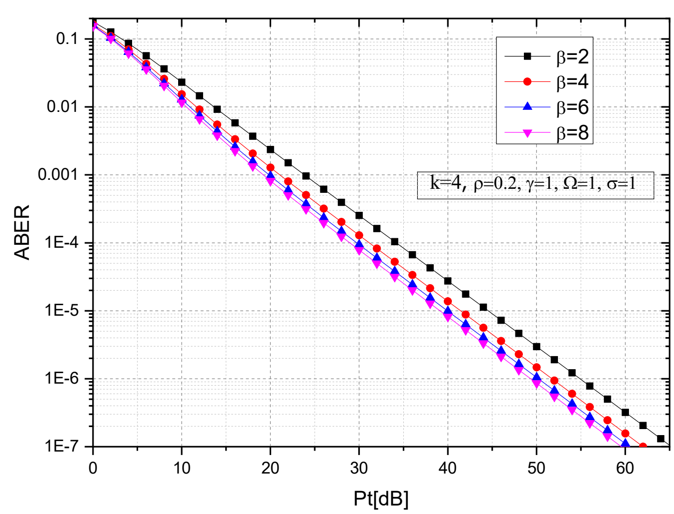

Fig. 3. ABER for the Shadowed Chi-square/Chi-square FSO channel model

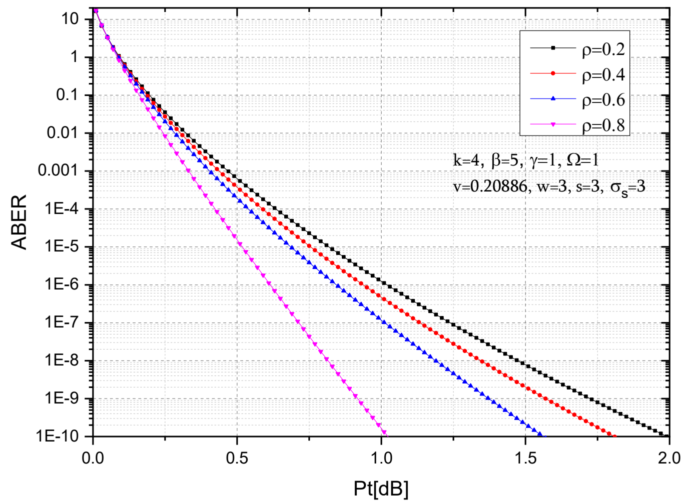

Fig. 4. ABER for the Shadowed Chi-square/Chi-square Boresight pointing error channel model.

6 Conclusion

In conclusion, this paper presents a novel general model for the irradiance fluctuations in Free Space Optical (FSO) systems based on the Chi-square distribution. The error performance of an FSO system modulated with On-Off Keying (OOK) scheme is investigated considering both atmospheric turbulence and misalignment fading (pointing error). The Average Bit Error Rate (ABER) expressions are derived in analytically closed form and analyzed in terms of the average optical power at the transmission. The graphical results demonstrate the impact of relevant parameters on the quality of the received signal in the OOK modulated FSO system. This research contributes to a better understanding of the effects of atmospheric turbulence and misalignment on FSO communication systems, enabling improved system design and performance optimization.

References

1. J. H. Churnside ,S.F. Clifford, Log-normal Rician probability-density function of opticalscintillations in the turbulent atmosphere, Journal of Optical Society of America A 4(10), 1987,pp. 1923–1930, DOI: 10.1364/JOSAA.4.001923.
2. E. Jakeman, P. Pusey, Significance of K distributions in scattering experiments, Physical Review Letters 40(9), 1978, pp. 546–550, DOI: 10.1103/PhysRevLett.40.546.

3. L. Andrews, R. Phillips, Mathematical genesis of the I-K distribution for random optical fields, Journal of Optical Society of America A 3(11), 1986, pp. 1912–1919, DOI:10.1364/JOSAA.3.001912.
4. M. A. Kashani, M. Uysal, M. Kavehrad, A novel statistical channel model for turbulence-induced fading in free-space optical systems, Journal of Lightwave Technology 33(11), 2015, pp.2303-2312, DOI: 10.1109/JLT.2015.2410695.
5. A. Jurado-Navas, J. M. Garrido-Balsells, J. F. Paris, A. Puerta-Notario, A unifying statistical model for atmospheric optical scintillation, arXiv preprint arXiv:1102.1915, 2011, DOI:10.5772/25097.
6. N. D. Chatzidiamantis, H. G. Sandalidis, G. K. Karagiannidis, S. A. Kotsopoulos, Matthaiou M., New results on turbulence modeling for free-space optical systems, Proceedings of 2010 17th IEEE International Conference on Telecommunications (ICT), April 4-7, 2010, Doha, Qatar, pp. 487–492.
7. J. H. Churnside, R. G. Frehlich, Experimental evaluation of log-normally modulated Ri-cianand IK models of optical scintillation in the atmosphere, Journal of the Optical Society of AmericaA 6(11), 1989, pp. 1760–1766, DOI: 10.1364/JOSAA.6.001760.
8. F. Yang, J. Cheng, Coherent free-space optical communications in Lognormal-Rician Tur-bulence, IEEE Communications Letters 16(11), 2012, pp. 1872–1875, DOI:10.1109/LCOMM.2012.100812.121341.
9. A. Belmonte, J. M. Kahn, Performance of synchronous optical receivers using atmospheric compensation techniques, Optics Express 16(18), 2008, pp. 14151–14162, DOI: 10.1364/OE.16.014151.
10. H. Zhou, W. Xie, L. Zhang L, Y. Bai, W. Wei, Y. Dong, Performance analysis of FSO coherent BPSK systems over Rician turbulence channel with pointing errors, Optics Express 27(19), 2019, pp. 27062-27075, DOI: 10.1364/OE.27.027062.
11. S. M. Aghajanzadeh, M. Uysal, Diversity-multiplexing trade-off in coherent free-space op-tical systems with multiple receivers, Journal of Optical Communications and Networking 2(12), 2010, pp. 1087– 1094, DOI: 10.1364/JOCN.2.001087.
12. S. Panić, H. Milošević, B. Prlinčević, Performance analysis of FSO transmission of double watermarked image over the Double Rician turbulence channel, Proceedings of First West Asian Colloquium on Optical Wireless Communications (WACOWC), April 25, 2018, Isfehan, Iran.
13. S. Panić, H. Milošević, S. Vasić, V. Milenković, Dynamical characteristics of the FSO trans-mission capacity in the presence of Rician turbulence, Proceedings of 2018 International Conference on Information and Communications Technology (ICOIACT), March 6-7, 2018, Jogyakarta, Indonesia.
14. B. P. Prlinčević, S. R. Panić, P. C. Spalević, M. A. Mišić, A. Amniesi, V. Stanojević, On the transmission of double watermarked image over Rician FSO channel, Elektronika ir Elektro-technika 22(3), 2016, pp. 83–88, DOI: 10.5755/j01.eie.22.3.15320.
15. M. A. Al-Habash, L. C. Andrews, R. L. Phillips, Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media, Optical Engineering 40(8), 2001, pp. 1554-1562, DOI: 10.1117/1.1386641.
16. P. Saxena, A. Mathur, M. R. Bhatnagar, BER performance of an optically pre-amplified FSO system under turbulence and pointing errors with ASE noise, Journal of Optical Com-munications and Networking 9(6), 2017, pp. 498-510, DOI: 10.1364/jocn.9.000498.
17. E. E. Elsayed, B. B. Yousif, Performance enhancement of the average spectral efficiency using an aperture averaging and spatial-coherence diversity based on the modified-PPM modulation for MISO FSO links, Optics Communications 463, 2020, DOI: 10.1016/j.opt-com.2020.125463.

18. M. Smilić, Z. Perić, D. Milić, A. Marković, M. Savić, Influence of zero and non-zero boresight pointing errors on bit-error rate free-space optic transmission over Málaga atmospheric channel, Optica Applicata 48(2), 2018, pp. 179–189, DOI: 10.5277/oa180202.
19. I. S. Ansari, M. S. Alouini, J. Cheng, Ergodic capacity analysis of free-space optical links with nonzero boresight pointing errors, IEEE Transactions on Wireless Communications 14(8), 2015, pp.4248–4264, DOI: 10.1109/TWC.2015.2418285.
20. H. G. Sandalidis, T. A. Tsiftsis, G. K. Karagiannidis, M. Uysal, BER performance of FSO links over strong atmospheric turbulence channels with pointing errors, IEEE Communications Letters 12(1), 2008, pp. 44-46, DOI: 10.1109/LCOMM.2008.071408.
21. H. G. Sandalidis, T. A. Tsiftsis, G.K. Karagiannidis, Optical wireless communications with heterodyne detection over turbulence channels with pointing errors, Journal of Lightwave Technology 27(20), 2009, pp. 4440-4445, DOI: 10.1109/JLT.2009.2024169.
22. N. Milošević, M. Petković, G. Djordjević, Average BER of SIM-DPSK FSO system with multiple receivers over M-distributed atmospheric channel with pointing errors, IEEE Photonics Journal 9(4), 2017, pp. 1-10, DOI: 10.1109/JPHOT.2017.2710320.
23. G. L. Stüber, Principles of mobile communication, 2nd Edition, Kluwer Academic Publishers, 2002.
24. I. S. Gradshteyn, I. M. Ryzhik, Table of integrals, Series, and Products, 7th Edition, Elsevier Academic Press, 2007.
25. G. T. Djordjević, M. I. Petković, M. Spasić, D. S. Antić, Outage capacity of FSO link with pointing errors and link blockage, Optics Express 24(1), 2016, pp. 219-230, DOI: 10.1364/oe.24.000219.
26. A. A. Farid, S. Hranilović, Outage capacity optimization for free-space optical links with pointing errors, Journal of Lightwave Technology 25(7), 2007, pp. 1702–1710, DOI: 10.1109/JLT.2007.899174.
27. J. Todorovic, P. Spalevic, S. Panic, B. Milosavljevic, M. Gligorijevic, "FSO system performance analysis based on novel Gamma – Chi-square irradiance PDF model", Optica Applicata, ISSN: 0078-5466 , DOI 10.37190/oa210303
28. The Wolfarm Functions Site: MeijerG functions. [Online] Available: http://functions.wolfram.com/PDF/MeijerG.pdf
29. Sawhil, P. Bhardwaj, Effect of atmospheric turbulence and pointing error on OOK in free space optics, International Journal of Engineering Trends and Technology (IJETT) 59(3), 2018, pp. 122-126.
30. H. Zhang, C. Li, C. Hao, Performance analysis for BPSK, DPSK and OOK-based FSO system in atmospheric turbulence conditions, International Journal of Simulation - Systems, Science & Technology 17(36), 2016, pp. 371-376, DOI: 10.5013/ IJSSST.a.17.36.37.
31. The Wolfarm Functions Site: Erfc functions. [Online] Available: http://functions.wolfram.com/PDF/Erfc.pdf
32. A. Abdi, W. C. Lau, M. S. Alouini, and M. Kaveh, "A new simple model for land mobile satellite channels: First and second order statistics," IEEE Trans. Wireless Commun., vol. 2, no. 3, pp. 519–528, May 2003