Using of Finite Element Method for Modeling of Mechanical Response of Cochlea and Organ of Corti

Velibor M. Isailovic, Milica M. Nikolic, Dalibor D. Nikolic, Igor B. Saveljic and Nenad D. Filipovic, Member, IEEE

Abstract—Human hearing system is very interesting for investigation. There are several parts in hearing system, but the most important parts in sense of conversion of audio signal in electrical impulse are cochlea and organ of Corti. The reason why scientists investigate mechanical behavior of human hearing system is hearing loss – a health problem that affects a large part of the world’s human population. That problem can be caused by aging, as consequence of mechanical injuries or some disease or even can be congenital. The experimental auditory measurements provide information only about the level of hearing loss, but without information what is happening in the hearing system. Therefore, it is very helpful to develop a numerical model of the parts of hearing system such as cochlea and organ of Corti to demonstrate process of conversion of acoustic signals into signals recognizable by human brain. Two numerical models are developed to investigate hearing problems: tapered three-dimensional cochlea model and two-dimensional cochlea cross-section model with organ of Corti.

I. INTRODUCTION

HUMAN hearing system consists of several parts: external auditory canal, tympanic membrane, three very small ossicles (malleus, incus and stapes) and cochlea [1], [2], [3]. The role of all parts situated before the cochlea is to transmit audio signals from the environment into the most important part of the inner ear – cochlea [4]. The role of cochlea is to transform mechanical vibrations into electrical impulses and send them via the cochlear nerve to the brain. The main mechanisms in the cochlea take place along the basilar membrane and the organ of Corti. Oscillations of the basilar membrane occur due to oscillations in the fluid chambers, which are transmitted through the middle ear. The organ of Corti, which contains an array of cells sensitive to basilar membrane vibration, lies at the surface of the basilar membrane. Those cells are known as outer and inner hair cells. They produce electrical signal under the influence of the basilar membrane.

For modeling the whole process, we have developed two different models. The first one is a three-dimensional tapered cochlea model which contains several parts: basilar membrane, fluid chambers (scala tympani and scala vestibuli), oval window, round window and outer shell. This model is used only to simulate response of basilar membrane. The second model is a two-dimensional slice model which is used to simulate the motion of all parts of the organ of Corti.

II. METHODS

The mathematical model for mechanical analysis of the behavior of the cochlea includes acoustic wave equation for fluid in the cochlear chambers and Newtonian dynamics equation for the solid part of the cochlea (vibrations of the basilar membrane) [5].

Acoustic wave equation is defined as:

$$\frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$$

where \( p \) stands for fluid pressure inside the chambers, \( x_i \) are spatial coordinates in Cartesian coordinate system, \( c \) is the speed of sound, and \( t \) is time.

Matrix form of the acoustic wave equation, obtained by using Galerkin method, can be presented in the following formulation:

$$Q \ddot{p} + H p = 0$$

where \( Q \) is the acoustic inertia matrix, and \( H \) represents the acoustic “stiffness” matrix.

The motion of the solid part of the cochlea was described by Newtonian dynamics equation:

$$M \ddot{U} + B \dot{U} + K U = F^{ext}$$

In equation (3), \( M \), \( B \) and \( K \) stands for mass, damping and stiffness matrix, respectively.

The real material properties of the basilar membrane are nonlinear and anisotropic. Also, dimensions of the cross-section of the basilar membrane are not constant along the cochlea. In order to match place-frequency mapping, the value of stiffness or geometry of the basilar membrane in finite element model should be variable along the cochlea. In this model tapered geometry of basilar membrane was used to obtain frequency mapping. The basilar membrane width increases and thickness decreases from the beginning to the end of membrane.

In the frequency analysis damping could be included using modal damping [5]. In that case, inside the stiffness matrix there is an imaginary part, so equation (3) could be written in the following form:

$$M \ddot{U} + K(1+i\eta) U = F^{ext}$$

where \( \eta \) is the hysteretic damping ratio.
The fluid-structure interaction with strong coupling was used for solving these equations. Strong coupling means that the solution of solid element in the contact with fluid has impact on the solution of fluid element. The coupling was achieved by the equalization of normal fluid pressure gradient with normal acceleration of solid element in the contact, as shown in the equation (5).

\[ n \cdot \nabla p = \rho n \cdot \ddot{u} \]  

(5)

For the mechanical model of the cochlea we defined a system of coupled equations:

\[
\begin{bmatrix}
M & 0 \\
-\rho \mathcal{R} & \mathcal{Q}
\end{bmatrix}
\begin{bmatrix}
\ddot{U} \\
\ddot{P}
\end{bmatrix}
+ \begin{bmatrix}
K(1+i\eta) - \omega^2 \mathcal{M} & -\mathcal{S} \\
-\rho \mathcal{R} & H - \omega^2 \mathcal{Q}
\end{bmatrix}
\begin{bmatrix}
U \\
P
\end{bmatrix}
= \begin{bmatrix}
F \\
q
\end{bmatrix}
\]  

(6)

where \( R \) and \( S \) are coupling matrices.

The solutions for displacement of the basilar membrane and pressure of fluid in the chambers were assumed in the following form:

\[ U = A_u \sin(\omega t + \alpha) \]
\[ P = A_p \sin(\omega t + \alpha) \]  

(7)

In equation (7), \( A_u \) and \( A_p \) represent amplitudes of displacement and pressure, respectively. The circular frequency is \( \omega \), \( t \) is time and \( \alpha \) is phase shift.

When displacement and pressure solution (7) were substituted in the equation (6) we obtain a system of linear equations that can be solved:

\[
\begin{bmatrix}
K(1+i\eta) - \omega^2 \mathcal{M} & -\mathcal{S} \\
-\rho \mathcal{R} & H - \omega^2 \mathcal{Q}
\end{bmatrix}
\begin{bmatrix}
A_u \\
A_p
\end{bmatrix}
= \begin{bmatrix}
0 \\
q
\end{bmatrix}
\]  

(8)

For solving the system of equations (8), in-house numerical program was developed. The program is part of PAK software package [7], [8].

III. FINITE ELEMENT MODELS

As already mentioned in the introduction, we have developed two different models. The first one is a three-dimensional tapered cochlea model. This model is used only for investigation of mechanical response of the cochlea. The length of the model is 35 mm, which corresponds to the real length of the cochlear chambers. The cross-section is square with 3 mm edge length at the beginning of basilar membrane and 1 mm edge length at the end of basilar membrane. Here is used orthotropic material model for modeling of basilar membrane. The material properties are given in Table 1. 3D model of the cochlea is given in Fig. 1.

The boundary condition in the fluid domain is prescribed acoustic pressure at round window, the beginning of upper fluid chamber. That excitation corresponds to reality when an audio signal comes into the hearing system. This signal is then transmitted through the elements of the middle ear to the cochlea. The unit value is prescribed because the value is not significant. This model is used only to analyze modal shapes.

![Fig. 1. 3D finite element model of the tapered cochlea](image)

The boundary conditions for the basilar membrane are clamped edges.

The third boundary condition is fluid-structure interface at all surfaces where fluid and solid finite elements are coupled face to face.

The second model that we use to model active cochlea is a 2D slice model (Fig. 2). This model is generated depending on excitation frequency in the 3D model. Several different excitation frequencies were investigated. For each specific frequency we solve the 3D model and determine the peak in basilar membrane response.
After that, we reconstruct the cross-section of the cochlea at that place and generate 2D slice model. The pressure at basilar membrane calculated by the 3D model is used as boundary condition. (Fig. 3).

IV. RESULTS

The response of the basilar membrane was obtained using the tapered three-dimensional model, Fig. 4. Here are presented the results for only one excitation frequency of 3450 Hz. The pressure distribution is obtained by using tapered cochlea model and after that is prescribed in a proper 2D slice model. In Fig. 5 contour plot for fluid pressure in 3D model and displacement field in 2D slice model are shown.

V. CONCLUSION

The processes that appear in human ears are very interesting for research. Medical doctors are mainly engaged in experimental research to measure the level of hearing damage. Their knowledge helps engineers to approximate the auditory system in an appropriate way, to avoid less important parts and to make modeling of the most important
parts in a right way, in order to obtain meaningful results.

ACKNOWLEDGMENT

This work was supported in part by grants from Serbian Ministry of Education and Science III41007, ON174028 and FP7 ICT SIFEM 600933.

REFERENCES