Distortion Optimized Spherical Cube Mapping for Discrete Global Grid Systems

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***Abstract*—The amount of geospatial data generated globally, together with the necessity for increased interoperability of these data, call for innovative solutions for global geospatial reference frames. Discrete Global Grid Systems are a class of spatial reference systems that use hierarchical tessellation of cells to partition and address the globe without gaps or overlaps. The specific properties of DGGS make them important candidates for future standard geospatial reference frames and fuel further efforts to investigate their potential for organization, exchange and processing such data. In this paper, we focus on DGGSs based on the so-called spherical cube mapping technique and present some first results of how these can be optimized to serve as global reference frames for large volumes of gridded geospatial data.**

I. INTRODUCTION

The amount of geospatial data increases with a very high velocity. As an example, high-resolution satellite and airborne footages are collected at rates of many terabytes per day. The organization of such data-collections becomes challenging per se, while processing and analyzing require suitable spatial reference frames. So far most satellite missions have come up with own individual reference grid(s) for global data representation. To facilitate the fusion of data from different missions, standardized multi-resolution reference frames would, however, be necessary.

One option to define a hierarchical tessellation of near equal-area cells at multiple levels of granularity for the entire Earth is called Discrete Global Grid Systems (DGGS) [1]. The importance of DGGS is underlined by the fact that the Open Geospatial Consortium (OGC) has founded the DGGS Standard and Domain Working Groups to foster the interoperability of geospatial data. A lot of DGGS has been proposed in recent years, with various methods achieving the proper tessellation of the surface [2]. Most of such systems are based on regular, multi-resolution partitions of polyhedra, called Geodesic DGGSs [3]. The two out of five design choices that fully specify a Geodesic DGGS, according to [3], are a base regular polyhedron and its orientation relative to the Earth.

A significant number of proposed DGGSs are based on the icosahedron and use triangular or hexagonal cells [2]. Despite their good properties in approximating the Earth's surface, the absence of orthogonal axes and cell congruency, as well as a complicated implementation seem to prevent their widespread acceptance. On the other hand, cube-based DGGSs introduce greater distortion, because of the lower number of primary partitions. However, the ease of the implementation and superior properties in data organization and retrieval make them more attractive for the usage in different applications. The main motivation for this paper is to boost the public interest for the application of the cube based DGGS by minimizing area distortion in the ellipsoid to sphere mapping and the distortion of the landmass projection through the orientation of the base cube.

II. RESEARCH QUESTIONS

The term Discrete Global Grid System is relatively new [2], but the need for a global system that would collect spatial data from all over the world is much older. Without better nomenclature, they were referred to as Earth database systems at that time. Some attempts to develop an Earth database system based on a Quadrilateralized Spherical Cube dates from the early 1970s [4]. The proposed system was modified later [5], and served for the Cosmic Background Explorer (COBE) project at NASA. Several decades later, cube-based DGGS regain popularity [6-7], mainly because they provide quadrilateral cells that can be efficiently handled [8].

Although there are numerous spherical cube map projections [9], most of the published papers about them deal with the properties of projecting the sphere to a cube, as their names imply. However, the implementation of DGGS requires the usage of a more accurate approximation of the Earth's surface, such as the WGS84 ellipsoid. This paper provides an answer to the question of what the properties of such projections are when the ellipsoid is projected to a cube and whether the distortions can be minimized by additional transformations.

The second question relates to a possibility to reduce the amount of distortion on the landmass if the projection cube is rotated, so that the areas with larger distortions are placed over the oceans or other water bodies.

The two previously described steps for the WGS84 ellipsoid projection to a cube are combined into a pipeline of transformations. These can, then, serve for constructing DGGSs that would provide effective solutions for the needed standardized reference frames, boosting interoperability of global raster data.

III. DISTORTION OPTIMIZATION

In this paper, we focus on the two aspects of distortion optimization: minimizing the influence of ellipsoid to sphere mapping and reducing distortion over certain areas by base polyhedron rotation. Although the greatest influence on the distortion comes from the chosen sphere to cube mapping, the principles described in this paper are generally applicable to all of them. To illustrate the impact of proposed methods, we have chosen adjusted spherical cube [10-11], as easy to implement, yet very efficient method to project sphere to a cube [9]. The forward transformation, i.e. mapping the spherical (φ,θ) to rectangular (x,y) coordinates of the cube face is defined by (1) and (2).

$$x = \varphi \cdot 4/\pi \tag{1}$$

$$y = \arctan(\ \tan(\theta) / \cos(\varphi)\ ) \cdot 4/\pi \tag{2}$$

The adjusted spherical cube mapping is neither equal-area, nor conformal projection, but the maximum-to-minimum area distortion is 1.4142, which is far better than most other non-equal-area spherical cube map projections, for which this parameter ranges from 2.0, for the Continious Cube mapping [12], up to 5.2 for the Tangential Spherical Cube [9].

A. Transformation Pipelines

In order to provide the WGS 84 to Cube-map coordinates transformation, and vice versa, proper pipelines are defined. Fig.1 depicts the main steps in both the forward and the inverse pipeline, combined to a cycle of coordinate transformations.

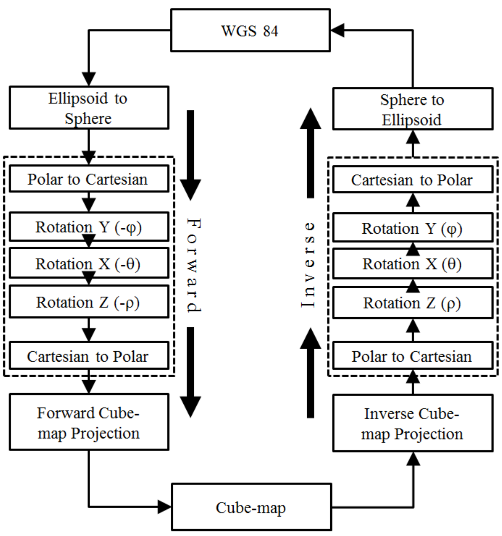

Figure 1. Coordinate transformation cycle consisting of the two pipelines – forward and inverse

The forward transformation starts with the ellipsoid to sphere transformation, presented in the next section. This step is optional and serves to reduce a certain type of distortion, according to the property we want to preserve.

The next step rotates the base cube, diminishing the distortion over the areas of interest. Since the distribution of the distortion is fixed across the faces of the cube, and depends on the chosen projection only, the impact on certain areas can be changed by rotating the base cube.

The last step in the forward pipeline is mandatory. It defines the actual sphere to cube transformation.

The inverse pipeline converts spherical cube map coordinates back to WGS84, consisting of the reverse order of the inverse transformations from the forward pipeline.

B. Ellipsoid to Shere Transformation

The ellipsoid to sphere transformation is the first stage in the forward pipeline. It is not a unique process and depends on the property that should be preserved. A common way to perform this step is to transform geodetic latitude to some "auxiliary" latitude.

The geodetic latitude is an angle between the equatorial plane and the vector perpendicular to the surface of the ellipsoid at a given point. It is slightly greater than any auxiliary latitude, except at the Equator and poles, where they are all equal. Spherical forms of map projections can be adapted for use with the ellipsoid by substituting the geodetic latitude with one of the various auxiliary latitudes. The auxiliary latitudes were systematically described and all formulas derived by O. Adams [13], in 1921, but wider popularity is gained much later with Snyder's working manual [14].

There are six auxiliary latitudes, each with certain properties:

- geocentric (ϕ) – an angle between the equatorial plane and the radius vector,
- parametric (η) – the parallel on the sphere (with the radius equal to a semi-major axis) has the same radius as the parallel of geodetic latitude,
- conformal (χ) – preserves angles,
- authalic (β) – preserves surface area,
- rectifying (μ) – preserves distances along meridians and
- isometric (ψ) – equal increments of isometric latitude and longitude correspond to equal distance displacements along meridians and parallels.

Geocentric and parametric latitudes are the simplest to compute. In both cases, the ratios of tangents of given auxiliary and geodetic latitude are constants. Rectifying latitude represents the other extreme on the calculation scale. It cannot be expressed in the closed-form and requires series or numerical integration. Isometric latitude is also specific. It rapidly diverges from the geodetic latitude, tending to infinity at the poles. Both, rectifying and isometric latitudes, are out of the scope of this paper.

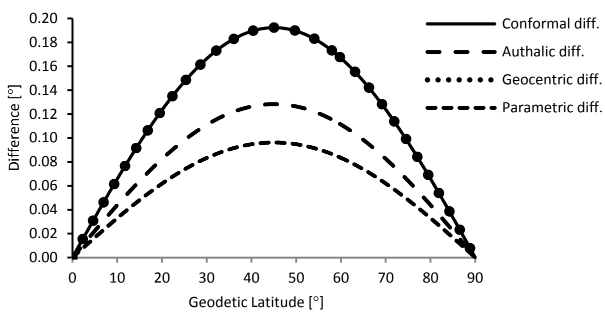

Figure 2. Auxiliary Latitudes – The divergence from the geodetic latitude

The divergence from the geodetic latitude of the four most frequently used auxiliary latitudes is shown in Fig.2. The difference is maximal at around 45°. It is interesting to notice that geocentric and conformal latitudes are almost the same; hence, being much easier to compute, the geocentric latitude usually substitutes the conformal latitude in the calculations.

One of the main problems in cartography is preserving sizes and shapes. However, as in the projecting a sphere to a plane, projecting an ellipsoid to a sphere cannot preserve both of them. The projection can be either a conformal or equal-area. For preserving angles, geocentric latitudes can be used as a good approximation; while preserving surface area requires application of authalic latitudes.

The authalic latitude (β) is very complex to compute and requires multiple iterations for the inverse transformation. The equations (3) through (5) define forward (geodetic to authalic), while (6) through (8) define inverse (authalic to geodetic) transformation.

$$\beta = \arcsin(q/q_p) \tag{3}$$

$$q = (1 - e^2)\ \{\sin\theta / (1 - e^2\sin^2\theta) - (1/(2e))\ \ln[(1 - e\sin\theta) / (1 + e\sin\theta)]\} \tag{4}$$

$$q_p = q_{(\theta=90°)} = (1 - e^2)\ \{1 / (1 - e^2) - (1/(2e))\ \ln[(1 - e) / (1 + e)]\} \tag{5}$$

$$q = q_p \sin\beta \tag{6}$$

$$\theta_0 = \arcsin(q/2) \tag{7}$$

$$\theta_{i+1} = \theta_i + [(1 - e^2\sin^2\theta_i)^2 / (2\cos\theta_i)]\ \{q/(1 - e^2) - \sin\theta_i / (1 - e^2\sin^2\theta_i) + (1/2e)\ln[(1 - e\sin\theta_i) / (1 + e\sin\theta_i)]\} \tag{8}$$

Aside from its complexity, the inverse authalic transformation loses its precision toward the poles, as shown in Fig.3. With a single iteration, the error is about 2 degrees at the pole, which corresponds to approximately 200km.

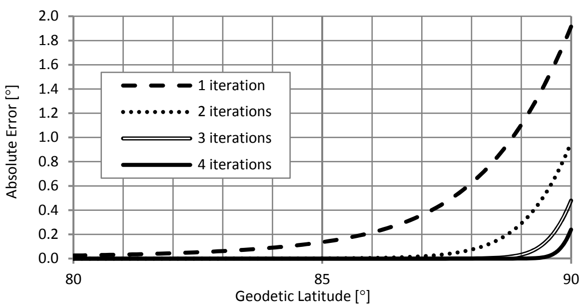

Figure 3. Authalic Latitude Inverse Function Error $|\theta - \mathrm{InvAuth}(\mathrm{Auth}(\theta))|$

Because of the very poor properties of the authalic latitude, like complex computation, iterative calculation of the inverse transformation with a loss of the precision in the proximity of the poles, a better solution is desirable. So, we propose an approximation, defined in (9), that is easy to compute, requires no iterations, and retains a very high precision throughout with a maximum deviation of about 0.1 arc-second (3m) around 25° latitude (see Fig.4).

$$\beta' = \arctan\,[(1 - e^2)^{2/3} \tan\theta] \tag{9}$$

Note that the approximated authalic latitude (9) has a form similar to the geocentric latitude ($\phi = \arctan\,[\,(1 - e^2) \tan\theta]$) and the parametric latitude ($\eta = \arctan\,[\,(1 - e^2)^{1/2} \tan\theta]$), with values somewhere in between the two.

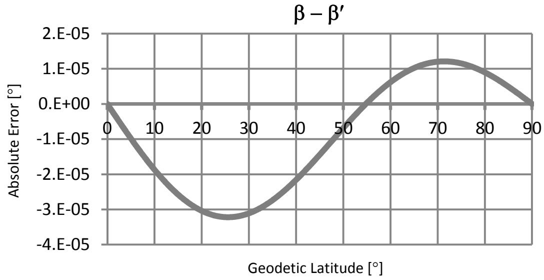

Figure 4. The difference between the authalic (β) and the approximated authalic (β') latitude

The impact of the chosen auxiliary latitude on the ellipsoid to sphere mapping distortion illustrated by the example of the adjusted spherical cube, is shown in section IV.

C. Base Cube Orientation

The distribution of the distortion depends on the chosen spherical cube map projection. Usually, the minimums are located at the centers of the faces, and the distortion increases toward the edges and corners of the base cube [9]. Hence, the impact on the area of interest can be diminished by rotating the cube and moving those areas toward the center of the faces.

The second phase in the forward pipeline performs the transformation by converting coordinates into the Cartesian coordinate system, rotating about all three axes, and transforming them back to the polar coordinate system.

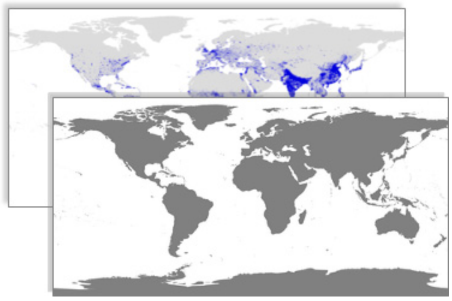

Figure 5. Raster masks used for the optimal base cube orientation

The optimal orientation can be found by varying rotation angles (φ, θ and ρ in Fig.1), from -45° to 45°, around all three axes, and comparing distortions over the areas of interest. These areas are confined by raster masks (Fig.5) defining landmass, population density, or any other criterion used for estimating an optimal orientation. The raster maps used as masks can be in any projection. However, for the sake of simplicity and efficiency, avoiding additional transformations, the maps used in experiments, as shown in Fig.5, are in LatLon WGS84 projection (EPSG:4326). The calculation is done for each pixel of all faces of the cube, that projects to a masked area, using the inverse pipeline. Since the calculation time is directly proportional to the resolution of the cube faces, the lower resolution is used for a wide range of angles, while higher resolution ones are used for fine-tuning of the base cube orientation, around expected extremes.

IV. Results and Discussion

A. *The Effects of Chosen Ellipsoid to Shere Transformation*

The two most frequently used metrics to depict shape deformations in the projection process are angular and areal distortions. Ideally, two lines should intersect at the same angle, both on the surface of the globe and on the projected map. If the projection is conformal, the angles are preserved and, hence, the shape of the features. For the non-conformal projections, the angular distortion represents the maximum deviation from the correct angle at a given location.

On the other hand, the projection also may alter the scale of the features. The ratio of the projected and original area is known as the areal distortion. Equal-area projections preserve the area. Unfortunately, the projection cannot be both conformal and equal-area. Preserving one feature leads to sacrifice of the other, and sometimes the distortion of the unpreserved feature can be severe. So, in most of the cases a compromise is required, and, hence, the projections that are neither conformal nor equal-area are very commonly used. The adjusted spherical cube is one of such projections.

Table I summarizes the effects of the distortion using different sphere-to-ellipsoid mappings. The first row contains parameters of the perfect sphere, while the next three contain distortion for the WGS84 ellipsoid using geodetic, geocentric and the approximated authalic latitude mappings, respectively. Each row is divided into three sub-rows, for the side face, top face and the cube as a whole (an averaged value for the four side faces and two top faces). The values are separately shown for the side and top faces to illustrate the asymmetry of the mappings. Table I does not contain the minimal value angular distortion column, since the value is always 0. The maximum-to-minimum is added as an additional column to the areal distortion, as it, probably, depicts the essential aspect of the surface preserving – a cell size variation across the surface of the map. Or, in our case, across the surface of the cube face.

TABLE I.
Distortion Effects of Various Sphere-to-Ellipsoid Mappings (Auxiliary Latitudes) on the Different Cube Faces

		Angular distortion		Areal distortion			
Type	Face	Max.	Avg.	Min.	Max.	Max./Min.	Avg.
Sphere	Side	31.085	11.569	1.621	2.293	1.414	1.925
	Top	31.085	11.569	1.621	2.293	1.414	1.925
	All	31.085	11.569	1.621	2.293	1.414	1.925
Geodetic	Side	30.962	11.570	1.632	2.308	1.414	1.934
	Top	31.332	11.588	1.610	2.293	1.424	1.921
	All	31.332	11.576	1.610	2.308	1.433	1.929
Geo-centric	Side	**31.085**	11.569	1.621	2.300	1.419	1.927
	Top	**31.084**	11.569	1.632	2.300	1.410	1.934
	All	**31.085**	11.569	1.621	2.300	1.419	1.929
Approx. authalic	Side	31.044	11.567	1.625	2.298	**1.414**	1.929
	Top	31.167	11.575	1.625	2.298	**1.414**	1.929
	All	31.167	11.570	1.625	2.298	**1.414**	1.929

As it is expected, geocentric latitude produces the smallest angular distortion, while approximated authalic produces the smallest area distortion. If geocentric latitude is used in ellipsoid to sphere mapping, there is an increase of about 0.34% in area distortion (max/min ratio), while angular distortion is kept at the level of a perfect sphere. On the other hand, approximated authalic latitude keeps the areal distortion; while maximum angular distortion is increased by 0.26%. The usage of the geodetic latitude yields the largest distortions:

- 0.8% the increase of maximum angular distortion,
- 0.056% the increase of average angular distortion and
- 1.35% the increase of area distortion (max/min ratio).

B. *The Optimal Orientation to Minimize Landmass Distortion*

There are lots of different criteria that can be used for choosing the best orientation of the base cube. One of the most prominent goals is to preserve continental plates of being split by the cube edges and reduce overall distortion of the landmass. Without rotations, all continents, except for Antarctica, are in quite unfavorable positions with regard to the cube faces, as shown in Fig. 7.

If the rotation angles are confined to integer numbers, the minimal angular distortion of the continental plates is gained for the following rotation angles: $\varphi = 17°$, $\theta = -10°$ and $\rho = 32°$. Fig. 6 illustrates the position of the base cube, after rotating by the defined angles.

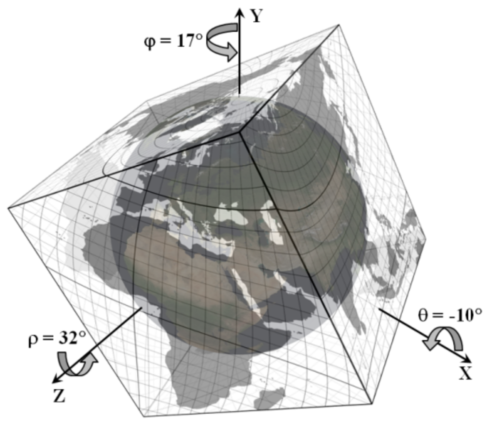

Figure 6. Optimal base cube orientation for the landmass distortion minimization

The rotation angles differ for minimum areal or aspect distortion, but we have chosen to minimize the angular distortion, because it has a wider range of possible values and hence a more noticeable difference between consecutive values of rotation angles. Also, the proposed rotation yields visually a very effective result, as can be seen in Fig. 8.

By using the proposed base cube orientation and approximated authalic latitude, an average angular distortion is reduced from 11.21° to 9.03°, while at the same time an average areal distortion is decreased from 1.92 to 1.86. Fig.8 shows the position of the continental plates on the cube faces for the optimal orientation of the base cube.

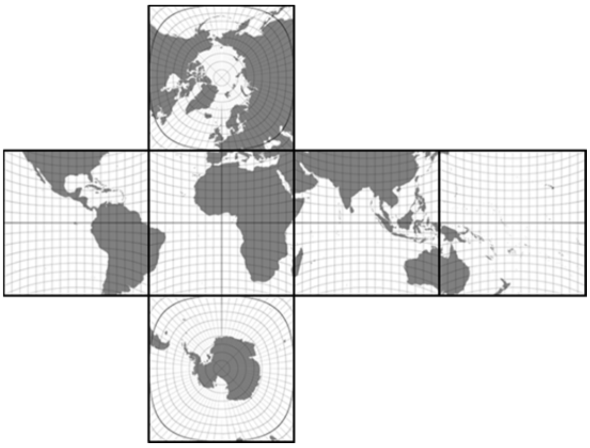

Figure 7. Cube-map faces for the initial base cube orientation (without rotations)

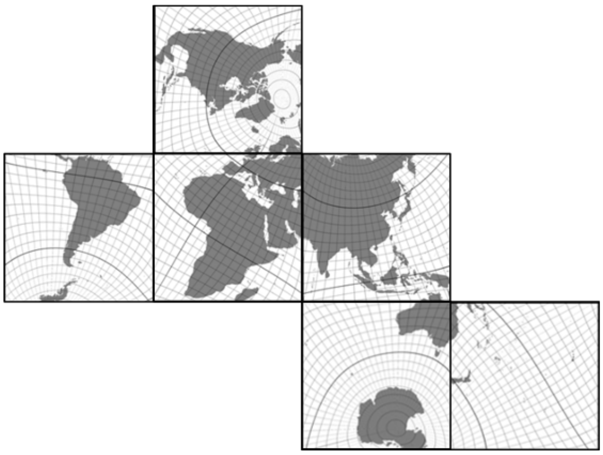

Figure 8. Cube-map faces for the optimal base cube orientation reducing landmass distortion

V. CONCLUSIONS

The need to store and organize large amounts of multiresolution geospatial data bring into play DGGS as a powerful concept. Geodetic DGGSs based on a cube, despite their relatively large distortion of the stored data, have a great potential to be accepted by a wide range of users due to the simplicity of implementation.

The effect of distortion can be reduced, to some extent, by choosing the appropriate mapping of the ellipsoid to the sphere and the orientation of the base cube. Given the almost spherical shape of the planet Earth, the choice of auxiliary latitude does not significantly affect the reduction of distortion imposed by ellipsoids to the sphere mapping. However, it is desirable to use the appropriate auxiliary latitude according to the type of projection, to preserve certain properties. For conformal projections, it is desirable to use geocentric latitude, while for equal-area projections it is desirable to choose authalic latitude. As the authalic latitude is very complex to compute, requires more iterations for the inverse transformation, and even with more iterations loses precision near the poles, an approximate function is proposed in this paper that eliminates all these shortcomings.

The orientation of the base cube cannot affect the overall distortion, but it can significantly reduce their impact on specific areas of interest. We have shown that by appropriate rotation the average angular distortion of continental plates can be reduced by almost 20% in the case of adjusted spherical cubes, while the area distortion is reduced by a much more modest 3%.

The proposed methods are part of the measures that should pave the way for enhanced DGGSs based on spherical cubes. Further research will be focused on other aspects of DGGSs, such as hierarchical spatial partitioning of cube pages, consideration of characteristics and efficiency of individual projections of spherical cubes, as well as finding an efficient method for visualization of such organized spatial data.

ACKNOWLEDGMENT

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