Topology Based Algorithm for Sectorisation of Water Distribution Network

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Abstract— In the field of Water Distribution Network (WDN) analysis, sectorisation is regarded as one of the main strategies for efficient management of WDNs. This paper presents an ICT-based solution which helps the managers to design the optimal number and size of sectors within the WDN. Presented algorithm relies on Graph Theory to search for the Strong Connected Components (SCCs) within the graph (water network), that are later topologically sorted and aggregated into sectors. Process of aggregation is driven by engineering criteria and relevant heuristics (e.g., sectors of approximately equal size with smallest number of links connecting them). Interventions in the network are not implemented in order to avoid negative effects on the networks’ hydraulics. This is important especially at primary stages of sectorisation in which preserving hydraulic performance and minimal investment are the main objectives. Methodology is illustrated on the real size WDN. Preliminary results show that new algorithm is efficient in terms of the computational speed and has the potential to be used inside an optimization algorithm where multiple alternatives and scenarios have to be investigated. Also, due to its robustness, algorithm can serve as an effective support tool for decision making process in engineering practice.

I. INTRODUCTION

Sectorisation or decomposition of a water distribution network (WDN) into zones (sectors, clusters or District Metered Areas - DMAs) has become one of the main strategies for efficient management of WDNs. Traditionally, sectorisation has been used in order to better control water losses in the network by means of observing all inflows and outflows from the sector. To accomplish this, network interventions such as installation of isolation valves and flow metering devices are required. However, if not implemented carefully, such interventions can significantly worsen the network supply reliability, water quality, fire-flow supply and system response in the case of accidental bursts and other failures. This is due to the fact that historically WDNs are developed as extremely looped systems in order to provide aforementioned requirements, and sectorisation into DMAs can considerably affect their topology. Complexity of the real life WDN results in many different alternatives in which network sectorisation can be done. Every WDN is unique in its topology and characteristics so there is no unique procedure for performing its’ sectorisation, but rather a series of guidelines provided by the different water authorities and used in this process by practice engineers.

Sectorisation is usually governed by the criteria of having sectors of “manageable size” in terms of number of consumers, links or network length, while network’s topology is often neglected. Widely used approach to sectorisation of a WDN used in engineering practice is “trial and error” method done by a local expert, which can lead to an arbitrary solution that can be far from (near) optimal one. Having in mind previously mentioned uniqueness of WDNs, there is a need for a practical tool that will enable greater involvement of engineering knowledge and specific limitations and constraints. For practical application in a real time decision making process, algorithm has to be fast to execute and robust.

In recent years different algorithms for automated sectorisation of the WDN into DMAs have been presented, as well as the tools that can be used to support this process [1, 2]. Majority of the presented methodologies are based on the Graph Theory algorithms [3, 4]. Others are using the modularity index [5] or community structure metrics [6] to perform the division of the WDN. Sectorisation algorithm is usually coupled with an optimization algorithm [5, 7] in order to search within broader solution space and ensure that a suboptimal feasible solution is identified. In sectorisation process different objective functions are used. For example, in [3] reachability of every potential solution is minimized, in [5, 6] modularity metrics [8] is maximized and in [7] minimization of dissipated hydraulic power is adopted. Resilience index, that regards to the network post-segmentation reliability [9], is often used as a main performance indicator [10] while in [6] fire-flow and water age metrics are added into consideration. However, despite all recent advancements made, scope exist to further improve existing water network sectorisation algorithms, especially in terms of usability for practicing engineers [11].

Methodology presented here is named Water Network Sectorisation (WNS) and is based on the heuristic aggregation of Strong Connected Components (SCCs) determined using the Graph Theory algorithms. Interventions in the network, such as closure of the valves or blockage of the pipes are not implemented at this point to avoid negative effects on networks’ hydraulics. The main goal of the WNS is to search for the optimal sectorisation suitable for the water balance control with minimal number of connecting links, implementing some engineering criteria and heuristics. Presented methodology can serve as a support tool for practice engineers when designing the sectorisation solution that will have the least effect on the hydraulics of the system and minimal investment. Furthermore, it can be used as a good starting point for narrowing down the solution space subjected to the optimization algorithm in search for the optimal solution. Also, possibility for hierarchical sectorisation should be investigated as it would enable perception of the WDN in the range of resolutions (level of detail)
depending on the task in consideration. For example, less detail resolution is required in primary stages of network analysis. Application of WNS algorithm will be illustrated on a real size water distribution network of a town in Serbia.

II. METHODOLOGY

The proposed Water Network Sectorisation (WNS) algorithm has three stages (Fig. 1) and hydraulic model of water distribution network is a prerequisite for its application.

In the first stage, hydraulic simulation of the analyzed WDN is performed to determine the orientation of pipes (based on water flow directions obtained in the simulation). Newly obtained graph is directed and in the Graph theory such graph is referred to as DIGRAPH.

In the second stage, parts of network in which the water is circulating during the simulation period (SCCs) are identified within the network.

Third stage covers the topological sorting of SCCs, identified in the 2nd stage, and their aggregation driven by the number of pre-specified engineering criteria. Although different algorithms from the Graph Theory could be used for topological sorting, the customized algorithm with some heuristics that bias the process is used here. Moreover, aggregation is done during the topological sorting by using pre-specified engineering criteria resulting in improved solutions identified. The following engineering criteria are used here: size of the sector in terms of water demand, number of connections between the sectors, pipe diameters and pipe lengths.

Finally, application of the algorithm results in identified sectors in the water distribution network.

In the following sections 2nd and 3rd algorithm stages will be explained in detail. For illustration purposes, a simple example network is used. This networks' digraph, which is result of the 1st algorithm stage, is shown in the Fig. 2a. Example network consists of 13 nodes and 19 links, where two of those links are identified as not oriented (7-8 and 2-3). Putting that in the context of water networks, those are usually pipes (links) that are connecting tanks with the rest of the network. So in the hypothetical simple network, nodes 8 and 2 could be tanks, and nodes 1 and 10 are the source nodes.

A. 2nd Stage

In the second stage SCCs are identified within the DIGRAPH, resulting in the formation of the Directional Acyclic Graph (DAG). Strong connected component (SCC) is a term from Graph Theory, and it is defined as a subgraph in which each node can be reached from any other node within that subgraph. Without using the terminology from Graph Theory, SCCs are parts of network in which the water is circulating during the simulation period. Due to that fact, control of the water balance and/or water pressure regulation in SCC parts of the network could be difficult to achieve, so the idea is to detect SCCs and treat them as aggregated nodes in further network analysis and clustering. Algorithms for the extraction of SCCs from digraph are well known in the Graph Theory [12]. Here, the Gabow algorithm (explained in [13]) is used. It requires only one pass through the network (DIGRAPH) with recursive call of the Depth First Search (DFS) algorithm with arbitrary selection of the starting node.

Starting the DFS search from the node 2, for the simple digraph shown in the Fig. 2a, nodes 3, 4, 6, 0 and 5 are visited. During the DFS propagation, a check is made weather the selection of the next node forms a cyclic path or not. If yes, nodes forming the cyclic path are identified as a SCC.

The algorithm continues until no further propagation is possible. In example in consideration, the first SCC component identified is composed of nodes 2, 3, 4, 6, 5 and 0. No further propagation is possible, so the DFS starts again from randomly selected node, chosen from the set of nodes that were not visited during the first search. Assuming that the randomly selected node is node 9, and after nodes 12 and 11 are visited, the second SCC composed of these three nodes is identified. DFS search is repeated again starting from node 8, and third SCC composed of nodes 8 and 7 is detected (Fig. 2b). At the end, aggregated digraph is composed of three identified SCCs. The digraph can also be viewed as set of

![Figure 2. A simple DIGRAPH transformation to DAG: a) Start the DFS, b) Detected SCCs, c) Newly formed DAG](image)

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aggregated nodes and two remaining nodes 1 and 10 (Fig. 2c). The most important property of new aggregated digraph is acyclicity, indicating it is a digraph without cycles. Such graph is referred to as Directed Acyclic Graph (DAG), and in terms of water network is very important, because it clearly separates source from the demand nodes and hence, makes the sectorisation of network easier.

B. Stage 3

Finally, the topological sorting of DAG and its aggregation by using a number of pre-specified engineering criteria (as explained below) is conducted, as shown in Fig. 3. The following engineering criteria are used here: size of the sector in terms of water demand, number of connections between the sectors, pipe diameters and pipe lengths.

Topological sorting of DAG starts from the most downstream (sink) nodes. Sink nodes are the one that have only inlet links, and propagation starts simultaneously from all these nodes. In the example discussed, SCC1 is the only sink node in the DAG, hence topological sorting will start from that node. Topological sorting continues in the direction opposite of the flow orientation, thus adjacent nodes upstream of SCC1 are candidates for propagation. A precondition for visiting an adjacent upstream node, and putting it in the set of signed (sorted) nodes, is that a candidate node has all of its downstream nodes already signed. This means that only nodes that have sorted (signed) adjacent downstream nodes are valid candidates for further upstream propagation and possible aggregation to the downstream component. Applying this precondition on the example DAG, the only valid candidate for propagation from SCC1 is SCC2, so it is signed and moved to the set of sorted nodes (Fig. 3a). Nodes 1 and 10 are source nodes (nodes without input links), so they are left for the final stage of algorithm.

The next step is to check if candidate node SCC2 could be aggregated with node SCC1. The primary criterion that is checked to determine if aggregation is feasible is the allowable size of the sector \((Q_{\text{max}})\) in terms of its demand. Maximum size for a sector is defined as \(Q_{\text{max}} = Q_{\text{tot}}/N_{\text{total}}\), where \(Q_{\text{tot}}\) is total input in the WDN and \(N_{\text{total}}\) is desired number of sectors, which has to be smaller than the number of identified SCCs. If the aggregated sector size is smaller than the \(Q_{\text{max}}\), then upstream node is joined to the downstream one. Furthermore, even if this criterion is not satisfied, nodes are aggregated if change in the sector size is smaller than the 10%. This is done to avoid the case where very small nodes are left as not aggregated. Following the simple example (Fig. 3), if we assume that all demand nodes have the same water consumption, node SCC2 will not be aggregated to node SCC1.

At this point, aggregation criteria have to be explained in more detail as the example in consideration does not illustrate complexity of real world networks. Unlike the Fig. 3 example, where only a single aggregation option exists (to aggregate SCC2 to SCC1 or not), in a real size network, a set of nodes that are valid candidates for propagation and aggregation according to the aforementioned primary criterion of maximum sector size is likely to exist. After checking that primary condition for aggregation, ranking of the remaining candidates is done according to the following additional criteria:

- Criterion 1: The number of links between the SCCs is checked. A candidate node whose aggregation would reduce the maximum number of links between the SCCs is chosen. Following this idea, aggregation of SCCs is done in such way that minimum number of links between SCCs remains at the end of algorithm run.

- Criterion 2: If multiple candidates exist after applying above criterion, a node is chosen by selecting the upstream link with the smallest diameter. This way the algorithm propagation through the main lines is postponed for the later stages. If several pipes with the same diameter exist, the pipe with minimum length is selected, in order to keep the sectors’ pipeline length minimal.

Going back to the simple example shown above, the next node feasible for propagation is node SCC3, simply because its’ adjacent downstream nodes SCC1 and SCC2 are already signed and topologically sorted. Next step is aggregation. Node SCC3 could be aggregated to SCC1 or SCC2. Given that SCC1 is already large enough and not feasible for further aggregation, node SCC3 is aggregated to SCC2 (Fig. 3b). The algorithm stops after the source nodes have been reached. As a result of the algorithm (Fig. 3c), there are 2 sectors – sector 1 made of nodes 0,2,3,4,5,6 (SCC1), and sector 2 made of nodes 7,8,9,11,12 (SCC2 and SCC3) – and 2 source nodes (1 and 10). There are five links connecting them.

Figure 3. DAG Aggregation: a) Upstream propagation and topological sorting; b) Components aggregation; c) Result of the algorithm

III. CASE STUDY

A. Description

The WNS algorithm presented here is applied to a real size WDN of a town in Serbia. This WDN supplies water to approximately 50,000 inhabitants, industry and public and commercial institutions. The water is pumped from wells into the Reservoir (Volume = 2 x 2500 m³), from where pumping station delivers clean water to the network. The WDN is divided into 3 zones based on
The center of town is divided into 2 sectors (SCT 1 and SCT 2), that are connected between themselves via 4 pipes. The most important connection is a pipe with a 500 mm diameter (location C), with maximum discharge that varies from about 100 to 153 l/s. Next two connection pipes have diameters of 200 and 80 mm (locations D and E, respectively), but with much smaller (almost negligible) discharges at the peak demand hour. In all three pipes flow direction is from SCT 2 to SCT 1. Fourth pipe connecting these two sectors has the diameter of 200 mm (location F) and is in the vicinity of the aforementioned pipe with the 80 mm diameter. The closure of this pipe is suggested due to the fact that simulated discharge was below 0.1 l/s, so measuring would not make any sense and closing it would not affect hydraulic performance of the network. Finally, three flow measuring locations (C, D and E) are proposed to control the flow from SCT 2 to SCT 1.

In order to complete the water balance in SCT 2, it is necessary to measure the water flow supplied to the sector from the Tank 1 (pipe with diameter 350 mm – location G), and water that is delivered from SCT 2 to SCT 5 through the pipes with diameters 300 mm and 125 mm (locations H and I, respectively). It should be also noted that one pipe between these two sectors with the 80 mm diameter is to be closed as well, due to the negligible flow (location J). This completes the water balance in the sectors SCT 2 and SCT 5 with additional 3 measuring locations (G, H and I).

Sector SCT 1 is supplied from SCT 2, and for the water balance control in SCT 1, it is necessary to measure water delivered to SCT 3. This is done through two pipes with the diameters of 200 and 80 mm (locations I and K, respectively), with one pipe with 100 mm diameter also being closed. Discharge that goes to Tank 1 from SCT 1 has to be calculated as well (location N). There is no need to establish a new measurement point for discharge at this location, simply because the water level in the Tank 1 and discharges from it (locations B and G) are already measured. In this manner, the water balance is fully controlled for the sectors SCT 1 and SCT 3 with 2 additional measuring locations. To fully control water balance in the sector SCT 4, supplied from the Tank 1, one more discharge measuring location is needed at the outlet of the small pumping station at Tank 2 supplying consumers in zone III (location O).

In summary, to control the water balance in the whole WDN it is necessary to establish 11 discharge measuring locations (see Fig. 4). Closure of 3 pipes with almost negligible discharges reduced the number of connecting links between the components to 11, from starting 14. It is obvious that some of the proposed measurement locations are suitable only for the water balance control, and cannot be used for reducing the pressures in the network. Those are the locations on water mains (600 and 500 mm) that deliver water to Tank 1. Other measurement locations could possibly be equipped with...
valves, and used for reducing pressure in sectors and consequently, water leakage.

IV. CONCLUSION

Efficient and simple algorithm for water network sectorisation (WNS) is presented in the paper. Algorithm uses results of 24-hour hydraulic simulation to determine the orientation of the pipes and to identify non-oriented pipes in which water flows in both directions during the simulation period. Once the oriented graph (DIGRAPH) is defined, SCCs are identified and aggregated to sectors according to the number of sectors set as an input. Algorithm is tailored to the need of finding the sectorisation solution that will have the least effect on the hydraulics of the system with minimal investment. The WNS algorithm was tested and demonstrated on the real-life network of a town in Serbia. Based on the results obtained, it can be concluded that the algorithm can serve as a valuable tool for practicing engineers dealing with water network sectorisation. Future work should be aimed at improvement of the algorithm by adding the possibility of testing the effects of potential network interventions. In this manner, different sectorisation solutions could be investigated through an optimization method and evaluated using some performance indicators, allowing selection of a (sub)optimal one.

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